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BHWIDE 1.00: $\mathcal{O}(\alpha)$ YFS Exponentiated Monte Carlo for Bhabha Scattering at Wide Angles for LEP1/SLC and LEP2[†]

S. Jadach
Institute of Nuclear Physics, ul. Kawiory 26a, Kraków, Poland
CERN, Theory Division, CH-1211 Geneva 23, Switzerland,
W. Płaczek*
Department of Physics and Astronomy,
The University of Tennessee, Knoxville, Tennessee 37996-1200
B.F.L. Ward
Department of Physics and Astronomy,
The University of Tennessee, Knoxville, Tennessee 37996-1200
SLAC, Stanford University, Stanford, California 94309
and
CERN, Theory Division, CH-1211 Geneva 23, Switzerland

Abstract

We present $\mathcal{O}(\alpha)$ YFS exponentiated results for wide angle Bhabha scattering at LEP/SLC energies using a new Monte Carlo event generator BHWIDE 1.xx. Our calculations include two options for the pure weak corrections, as presented in Beenakker et al. and in Böhm et al. From comparison with the results of Beenakker et al., Montagna et al. and Cacciari et al., we conclude that the total precision of our BHWIDE results is 0.3%(0.5%) in the LEP1/SLC regime within ± 100 MeV ($+2.75/-2.5$ GeV) of the Z peak. For LEP2, the corresponding precision is currently estimated at 1.5%; the latter could be improved if the data in LEP2 so require. Both precision tags represent clear improvements over what is currently available in the literature.

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1 Introduction

As the final LEP1 data analysis begins and the initial stages of LEP2 materialize while the SLD prepares for the beginning of what may be its final phase, the subject of precision calculations of wide angle Bhabha scattering becomes more and more interesting, particularly from the standpoint of realizing such calculations via a Monte Carlo event generator which would allow comparison between theory and experiment at the level of events in the presence of *arbitrary* detector cuts. Indeed, currently, the work of Beenakker et al. [1] and of Montagna et al. [2] represent the state of the art in the theoretical arena of the prespective calculations, and neither of these approaches has provided a genuine Monte Carlo event generator to allow comparison with arbitrarily cut experimental data. This has had the effect that the important Z physics parameter $\Gamma_{e\bar{e}}$ has been extracted from the data using a subtraction of the contamination from the t-channel γ-exchange in the wide angle acceptance relevant to its measurement. A Monte Carlo event generator with sufficient precision would obviate the need for this procedure, for example. Motivated by this and other such Z and LEP2 applications, we have developed an $\mathcal{O}(\alpha)$ Yennie-Frautschi-Suura (YFS) [3] exponentiated Monte Carlo event generator for wide angle Bhabha scattering at LEP1/SLC and LEP2 energies. We present this development in what follows.

Specifically, our starting point will be the $\mathcal{O}(\alpha)$ YFS exponentiated Monte Carlo (MC) event generator BHLUMI 1.xx developed by two of us (S.J. and B.F.L.W.) in ref. [4] for low angle Bhabha scattering in the SLC/LEP luminosity regime $17\,mrad < \theta_{e,\bar{e}} < 150\,mrad$, where $\theta_{e,\bar{e}}$ are the respective $e, \bar{e}$ CMS scattering angles. In order to arrive at an event generator valid for wide angles at both LEP1/SLC and at LEP2 to sufficient accuracy, we have had to introduce the effects of the Z exchange graphs into the calculations presented in ref. [4] and we have had to introduce the effects of the pure weak one-loop corrections into those calculations as well. We stress that we have made contact with the pioneering $\mathcal{O}(\alpha)$ MC program BABAMC of refs. [5, 6] as well as with the semi-analytical program ALIBABA of ref. [1] in that our pure weak corrections libraries are taken from these two cases: the user of our new wide angle Bhabha MC, BHWIDE 1.xx, may choose which library he uses and we shall discuss both of them in what follows. Our exact hard bremsstrahlung amplitude, in the presence of the full set of s-channel and t-channel γ and Z exchanges, we shall compute explicitly in what follows, using methods of the CALKUL-type [7], as formulated by[1] Xu et al. [9]. It is in this way that we have arrived at our new $\mathcal{O}(\alpha)$ YFS exponentiated MC event generator BHWIDE 1.xx which simulates realistic multiple photon radiative effects for wide angle Bhabha scattering in the LEP1/SLC and LEP2 energy regimes.

We have compared our results with those of BABAMC, of the Monte Carlo integrator program SABSPV of ref. [10], of the semi-analytical program TOPAZ0 of refs. [11, 12], of ALIBABA, of the MC event generator BHAGENE3 of refs. [13, 14], of the MC event generator BHAGEN95 of refs. [15, 16, 17, 18], and of the MC event generator UNIBAB of ref. [19]. On this basis, we have arrived at a reliable estimate of the total precision of our

[1] We note that Kleiss and Stirling [8] have introduced analogous methods as well.

results. The basis of this estimate is presented in detail in ref. [20] and we will review only its main features and the main features of the aforementioned attendant comparisons in what follows.

More precisely, our BHLUMI 1.xx Monte Carlo event generator realizes the process

$$e^+(p_1) + e^-(q_1) \longrightarrow e^+(p_2) + e^-(q_2) \ + \gamma_1(k_1) + \ldots + \gamma_n(k_n) \tag{1}$$

via the YFS exponentiated cross section formula

$$d\sigma = e^{2\alpha\, Re\, B + 2\alpha\,\tilde{B}} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j^0} \int \frac{d^4 y}{(2\pi)^4} e^{iy(p_1+q_1-p_2-q_2-\sum_j k_j)+D} \bar{\beta}_n(k_1,\ldots,k_n) \frac{d^3 p_2\, d^3 q_2}{p_2^0 q_2^0}, \tag{2}$$

where the real infrared function $\tilde{B}$ and the virtual infrared function B are given in refs. [3, 4, 21, 22, 23] (for definiteness, we record them in the form we shall use presently, as this representation is not readily available in the current literature and is essential for practical applications of the type we pursue here), and where we note the usual connections

$$\begin{aligned} 2\alpha\,\tilde{B} &= \int^{k\le K_{max}} \frac{d^3 k}{k_0} \tilde{S}(k), \\ D &= \int d^3 k \frac{\tilde{S}(k)}{k^0} \left(e^{-iy\cdot k} - \theta(K_{max} - k) \right) \end{aligned} \tag{3}$$

for the standard YFS infrared emission factor

$$\tilde{S}(k) = \frac{\alpha}{4\pi^2} \left[Q_f Q_{f'} \left(\frac{p_1}{p_1 \cdot k} - \frac{q_1}{q_1 \cdot k} \right)^2 + \ldots \right], \tag{4}$$

if Q_f is the electric charge of f in units of the positron charge. Here, the "$\ldots$" represent the remaining terms in $\tilde{S}(k)$ obtained from the one given by respective substitutions of Q_f, p_1, $Q_{f'}$, q_1 with corresponding values for the other pairs of the respective external charged legs according to the YFS prescription in ref. [3] (wherein due attention is taken to obtain the correct relative sign of each of the terms in $\tilde{S}(k)$ according to this latter prescription) and in refs. [24, 25], $f \neq e$, $f' = \bar{f}$. We have explicitly the representations

$$\begin{aligned} 2\alpha\Re B(p_1,q_1,p_2,q_2) + 2\alpha\tilde{B}(p_1,q_1,p_2,q_2;k_m) &= R_1(p_1,q_1;k_m) + R_1(p_2,q_2;k_m) \\ &+ R_2(p_1,p_2;k_m) + R_2(q_1,q_2;k_m) - R_2(p_1,q_2;k_m) - R_2(q_1,p_2;k_m) \end{aligned} \tag{5}$$

with

$$\begin{aligned} R_1(p,q;k_m) &= R_2(p,q;k_m) + \left(\frac{\alpha}{\pi}\right) \frac{\pi^2}{2}, \\ R_2(p,q;k_m) &= \frac{\alpha}{\pi} \Bigg\{ \left(\ln \frac{2pq}{m_e^2} - 1 \right) \ln \frac{k_m^2}{p^0 q^0} + \frac{1}{2} \ln \frac{2pq}{m_e^2} - \frac{1}{2} \ln^2 \frac{p^0}{q^0} - \frac{1}{4} \ln^2 \frac{(\Delta+\delta)^2}{4p^0 q^0} \end{aligned} \tag{6}$$

$$- \frac{1}{4}\ln^2\frac{(\Delta-\delta)^2}{4p^0q^0} - \Re Li_2\left(\frac{\Delta+\omega}{\Delta+\delta}\right) - \Re Li_2\left(\frac{\Delta+\omega}{\Delta-\delta}\right)$$
$$- \Re Li_2\left(\frac{\Delta-\omega}{\Delta+\delta}\right) - \Re Li_2\left(\frac{\Delta-\omega}{\Delta-\delta}\right) + \frac{\pi^2}{3} - 1\Bigg\}, \tag{7}$$

where $\Delta = \sqrt{2pq + (p^0 - q^0)^2}$, $\omega = p^0 + q^0$, $\delta = p^0 - q^0$, and k_m is a soft photon cut-off in the CMS ($E_\gamma^{soft} < k_m \ll E_{beam}$).

The YFS hard photon residuals $\bar{\beta}_i$ in (2), $i = 0, 1$, are given in ref. [4] for BH-LUMI 1.xx so that this latter event generator calculates the YFS exponentiated exact $\mathcal{O}(\alpha)$ cross section for $e^+e^- \to e\bar{e} + n(\gamma)$ with multiple initial, initial-final, and final state radiation using a corresponding Monte Carlo realization of (2) in the low angle regime of the SLC/LEP luminosity monitor acceptance, where the $e^\pm$ scattering angles $\theta_{e^\pm}$ satisfy $17\,mrad < \theta_{e^\pm} < 150\,mrad$. In the next sections, we use explicit Feynman diagrammatic methods and the results in refs. [1, 5, 6] to develop the corresponding Monte Carlo realization of the respective application of (2) to the wide angle scattering regime, $\theta_{e^\pm} \geq 150\,mrad$, for the Bhabha process $e^+e^- \to e^+e^- + n(\gamma)$.

We turn first to the implementation of the required wide angle physics effects in the YFS hard photon residual $\bar{\beta}_0$ through $\mathcal{O}(\alpha)$. We follow this discussion with the corresponding analysis for the wide angle physics effects in the hard photon YFS residual $\bar{\beta}_1$. In this way, we arrive at an exact $\mathcal{O}(\alpha)$ YFS exponentiated MC event generator valid for wide angle Bhabha scattering in the LEP1/SLC and LEP2 energy regimes.

2 Born and $\mathcal{O}(\alpha)$ contributions to $\bar{\beta}_0$ for wide angle Bhabha scattering at high energies

In this section we develop the Born and $\mathcal{O}(\alpha)$ contributions to the YFS hard photon residual $\bar{\beta}_0$ for the wide angle Bhabha scattering process. We use the low angle limit of the residual already presented in ref. [4] as a starting point and as a limiting case cross-check.

Specifically, the hard photon residual $\bar{\beta}_0$ is defined as follows through $\mathcal{O}(\alpha)$:

$$\frac{1}{2}\bar{\beta}_0 = \frac{d\sigma^{1-loop}}{d\Omega} - 2\alpha\Re B\,\frac{d\sigma_{Born}}{d\Omega}, \tag{8}$$

where $d\sigma^{1-loop}/d\Omega$ is the differential cross section for wide angle Bhabha scattering computed through the 1-loop correction and $d\sigma_{Born}/d\Omega$ is the respective Born differential cross section. In ref. [4], the right-hand side of (8) was computed in the low angle regime. Here, we need to compute (8) in the wide angle scattering regime. We do this as follows. First, we note that the exact expression for the wide angle scattering Born differential cross section in (8) is well-known

$$\frac{d\sigma_{Born}}{d\Omega} = \frac{1}{64\pi^2 s}\,|\mathcal{M}_0|^2, \tag{9}$$

where $|\mathcal{M}_0|^2$ is a spin averaged lowest order matrix element squared, given e.g. in eq. (22). Secondly, the complete 1-loop corrected wide angle Bhabha scattering differential cross section in (8) is also known and we use two different versions of it in our work, one taken from ref. [5] and one taken from ref. [1], where it is generally accepted that the latter version, which is the more recent of the two, is in fact the more up-to-date of the two [20] (and hence it is the option which we illustrate in our comparisons in Sect. 4). In our Monte Carlo event generator these two versions for $d\sigma^{1-loop}/d\Omega$ correspond to two choices for our electroweak library module, one of which the user specifies in his input file [26]. It is this way that we have realized the wide angle Bhabha scattering YFS hard photon residual $\bar{\beta}_0$ through $\mathcal{O}(\alpha)$.

We turn next to the wide angle Bhabha scattering YFS hard photon residual $\bar{\beta}_1$. This we do in the following section.

3 The YFS hard photon residual $\bar{\beta}_1$ for wide angle Bhabha scattering at high energies

In this section we present our determination of the $\mathcal{O}(\alpha)$ YFS hard photon residual $\bar{\beta}_1$ for wide angle Bhabha scattering at high energies. We start with the defining equation for $\bar{\beta}_1$.

The hard photon residual $\bar{\beta}_1$, to $\mathcal{O}(\alpha)$, is defined as follows

$$\frac{1}{2}\bar{\beta}_1 = \frac{d\sigma^{B1}}{kdkd\Omega_\gamma d\Omega} - \tilde{S}(k)\,\frac{d\sigma_{Born}}{d\Omega}, \tag{10}$$

where $d\sigma^{B1}/kdkd\Omega_\gamma d\Omega$ is the respective $\mathcal{O}(\alpha)$ bremsstrahlung differential cross section into the solid angles $d\Omega_\gamma$ and $d\Omega$ for the photon and positron respectively when the photon energy lies between k and $k+dk$. Thus, to specify $\bar{\beta}_1$ we need to specify the hard bremsstrahlung differential cross section on the RHS of (10). We now turn to this.

Specifically, the hard bremsstrahlung differential cross section required in (10) can be obtained via the standard methods from the corresponding helicity amplitudes $\mathcal{M}(\lambda_{e^+},\lambda_{e^-},\lambda'_{e^+},\lambda'_{e^-},\lambda_\gamma)$, where $\lambda_f^{(\prime)}$, $f=e^+,e^-$ is the incoming (outgoing) fermion helicity and λ_γ is the photon helicity. We have

$$\frac{d\sigma^{B1}}{kdkd\Omega_\gamma d\Omega} = \frac{1}{512\pi^5 s}\,\frac{(p_2^0)^2}{2E_b(E_b-k)}\,|\mathcal{M}|^2, \tag{11}$$

with

$$|\mathcal{M}|^2 = \frac{1}{4}\sum_{\substack{\lambda_i,\lambda'_j\\ i=e^\pm,\gamma,j=e^\pm}} |\mathcal{M}(\lambda_{e^+},\lambda_{e^-},\lambda'_{e^+},\lambda'_{e^-},\lambda_\gamma)|^2 \tag{12}$$

being the spin averaged squared matrix element. The formula (11) is given in the CMS of the incoming beams, where E_b is the beam energy. Thus, our determination of $\bar{\beta}_1$ will be completely specified when we give our results for $\mathcal{M}(\lambda_{e^+}, \lambda_{e^-}, \lambda'_{e^+}, \lambda'_{e^-}, \lambda_\gamma)$. We now turn to this.

More precisely, using the methods of ref. [9], we get the following representation for the non-vanishing helicity amplitudes:

$$
\begin{aligned}
M_1 &\equiv M(+++++) = -c\,s\,F_1\left[\frac{1}{t_p}R(t_p, a_La_R)\,G_1 + \frac{1}{t_q}R(t_q, a_La_R)\,G_2\right] \\
M_2 &\equiv M(++++-) = -c\,s'F_2^*\left[\frac{1}{t_p}R(t_p, a_La_R)\,G_1^* + \frac{1}{t_q}R(t_q, a_La_R)\,G_2^*\right] \\
M_3 &\equiv M(----+) = -c\,s'F_2\left[\frac{1}{t_p}R(t_p, a_La_R)\,G_1 + \frac{1}{t_q}R(t_q, a_La_R)\,G_2\right] \\
M_4 &\equiv M(-----) = -c\,s\,F_1^*\left[\frac{1}{t_p}R(t_p, a_La_R)\,G_1^* + \frac{1}{t_q}R(t_q, a_La_R)\,G_2^*\right] \\
M_5 &\equiv M(-++-+) = -c\,t_qF_4\left[\frac{1}{s'}R(s', a_La_R)\,G_3 + \frac{1}{s}R(s, a_La_R)\,G_4\right] \\
M_6 &\equiv M(-++--) = -c\,t_pF_3^*\left[\frac{1}{s'}R(s', a_La_R)\,G_3^* + \frac{1}{s}R(s, a_La_R)\,G_4^*\right] \\
M_7 &\equiv M(+--++) = -c\,t_pF_3\left[\frac{1}{s'}R(s', a_La_R)\,G_3 + \frac{1}{s}R(s, a_La_R)\,G_4\right] \\
M_8 &\equiv M(+--+-) = -c\,t_qF_4^*\left[\frac{1}{s'}R(s', a_La_R)\,G_3^* + \frac{1}{s}R(s, a_La_R)\,G_4^*\right] \\
M_9 &\equiv M(+-+-+) = +c\,u\,F_5\left[\frac{1}{t_p}R(t_p, a_L^2)\,G_1 + \frac{1}{t_q}R(t_q, a_L^2)\,G_2\right. \\
&\qquad\left. + \frac{1}{s'}R(s', a_L^2)\,G_3 + \frac{1}{s}R(s, a_L^2)\,G_4\right] \\
M_{10} &\equiv M(+-+--) = +c\,u'F_6^*\left[\frac{1}{t_p}R(t_p, a_L^2)\,G_1^* + \frac{1}{t_q}R(t_q, a_L^2)\,G_2^*\right. \\
&\qquad\left. + \frac{1}{s'}R(s', a_L^2)\,G_3^* + \frac{1}{s}R(s, a_L^2)\,G_4^*\right] \\
M_{11} &\equiv M(-+-++) = +c\,u'F_6\left[\frac{1}{t_p}R(t_p, a_R^2)\,G_1 + \frac{1}{t_q}R(t_q, a_R^2)\,G_2\right. \\
&\qquad\left. + \frac{1}{s'}R(s', a_R^2)\,G_3 + \frac{1}{s}R(s, a_R^2)\,G_4\right] \\
M_{12} &\equiv M(-+-+-) = +c\,u\,F_5^*\left[\frac{1}{t_p}R(t_p, a_R^2)\,G_1^* + \frac{1}{t_q}R(t_q, a_R^2)\,G_2^*\right. \\
&\qquad\left. + \frac{1}{s'}R(s', a_R^2)\,G_3^* + \frac{1}{s}R(s, a_R^2)\,G_4^*\right] \qquad (13)
\end{aligned}
$$

where our notation is as follows

- Invariants:

$$
\begin{aligned}
s &= 2p_1q_1, & s' &= 2p_2q_2,\\
t_p &= -2p_1p_2, & t_q &= -2q_1q_2,\\
u &= -2p_1q_2, & u' &= -2q_1p_2.
\end{aligned}
\tag{14}
$$

- Electroweak couplings:

$$
a_L = \frac{G}{e}(v_e + a_e), \quad a_R = \frac{G}{e}(v_e - a_e), \tag{15}
$$

where

$$
G = \frac{1}{\sin\theta_W \cos\theta_W}, \quad a_e = \frac{-1}{4\sin\theta_W\cos\theta_W}, \quad v_e = a_e(1 - 4\sin^2\theta_W). \tag{16}
$$

- Spinor functions:

$$
\begin{aligned}
F(p,q) &= \frac{< pq >}{< pq >^*},\\
G(p,q,r,s;k) &= \frac{< pq >^*}{< rk >< ks >},
\end{aligned}
\tag{17}
$$

where p, q, r, s denote fermion four-momenta in the massless limit, while k denotes the photon four-momentum, and the * stands for complex conjugation. Spinor products $< .. >$ are defined as

$$
< pq > = \sqrt{p_- q_+}\, e^{i\phi_p} - \sqrt{p_+ q_-}\, e^{i\phi_q}, \tag{18}
$$

where $p_\pm = p_0 \pm p_z$, $p_\perp = p_x + ip_y = \sqrt{p_+p_-}\, e^{i\phi_p}$.

- Propagator factor:

$$
R(x,y) = 1 + \frac{xy}{x - M_Z^2 + i\theta(x) M_Z \Gamma_Z}. \tag{19}
$$

- Specific notation:

$$
\begin{aligned}
&F_1 = F(p_1,q_1), \;\; F_2 = F(p_2,q_2), \;\; F_3 = F(p_1,p_2),\\
&F_4 = F(q_1,q_2), \;\; F_5 = F(p_1,q_2), \;\; F_6 = F(q_1,p_2),\\
&G_1 = G(p_1,p_2,q_1,q_2;k), \;\; G_2 = G(q_2,q_1,p_1,p_2;k),\\
&G_3 = G(q_2,p_2,p_1,q_1;k), \;\; G_4 = G(p_1,q_1,p_2,q_2;k),\\
&c = i2\sqrt{2}e^3.
\end{aligned}
\tag{20}
$$

The above helicity amplitudes have been obtained in the massless fermion approximation. However, to get a precise description of the photon radiation over the whole phase space (particularly for collinear configurations) the finite fermion masses have to be taken into account. This can be accomplished by adding to the matrix element $|\mathcal{M}|^2$ of eq. (12) the mass correction term [7]

$$\delta|\mathcal{M}|^2_{mc} = -e^2\left[\frac{m_e^2}{(kp_1)^2}|\mathcal{M}_0(s',t_q,u')|^2 + \frac{m_e^2}{(kq_1)^2}|\mathcal{M}_0(s',t_p,u)|^2 + \frac{m_e^2}{(kp_2)^2}|\mathcal{M}_0(s,t_q,u)|^2 + \frac{m_e^2}{(kq_2)^2}|\mathcal{M}_0(s,t_p,u')|^2\right], \tag{21}$$

where

$$|\mathcal{M}_0(s,t,u)|^2 = e^4\left\{\frac{1}{s^2}\left[|R(s,a_L^2)|^2u^2 + |R(s,a_R^2)|^2u^2 + 2|R(s,a_La_R)|^2t^2\right] + \frac{1}{t^2}\left[|R(t,a_L^2)|^2u^2 + |R(t,a_R^2)|^2u^2 + 2|R(t,a_La_R)|^2s^2\right] + \frac{1}{st}\,2\Re\left[R^*(s,a_L^2)R(t,a_L^2) + R^*(s,a_R^2)R(t,a_R^2)\right]\right\} \tag{22}$$

is the lowest order matrix element.

We have also used the results of ref. [7] for the matrix element $|\mathcal{M}|^2$ as a cross check and we have found that the two sets of results are in very good agreement with one another, well below the desired technical precision of 0.01% of our analysis for example. We also note that an equivalent representation of the results in ref. [7] has been given by Kleiss in ref. [27]. On introducing these two sets of results into the formula (10) for $\bar{\beta}_1$ and implementing the resulting expression into our YFS Monte Carlo program for Bhabha scattering, we arrive at the Monte Carlo event generator BHWIDE 1.00 for wide angle Bhabha scattering at high energies[2].

We will now illustrate the application of BHWIDE 1.00 to LEP1/SLC and LEP2 physics scenarios in the next section.

4 Results and comparisons

In this section, we present sample Monte Carlo data which we use to compare our predictions from BHWIDE 1.00 to those of related calculations as reported in ref. [20]. We discuss both LEP1/SLC energies and LEP2 energies.

Specifically, for our comparisons we use the same event selection (ES) cuts as defined in ref. [20]:

[2]We need to stress that, in implementing the results (11)–(22) and the corresponding results from ref. [7] into (10), care must be taken to compute the two terms on the RHS of (11) in the same manner with the same massive (massless) limits of the corresponding parts of each respective term so that the result for $\bar{\beta}_1$ is numerically stable; we have done this.

No.	E_{CM}	TOPAZ0	BHWIDE	BHAGENE3	ALIBABA	BHAGEN95
	(a) BARE $acol_{max} = 10^o$					
1.	88.45	$0.4579 \pm .0003$	$0.4560 \pm .0004$	$0.4495 \pm .0016$	$0.4575 \pm .0003$	$0.4578 \pm .0002$
2.	89.45	$0.6452 \pm .0002$	$0.6429 \pm .0006$	$0.6334 \pm .0023$	$0.6440 \pm .0003$	$0.6445 \pm .0003$
3.	90.20	$0.9115 \pm .0002$	$0.9087 \pm .0008$	$0.8997 \pm .0033$	$0.9090 \pm .0004$	$0.9095 \pm .0004$
4.	91.19	$1.1846 \pm .0002$	$1.1797 \pm .0010$	$1.1847 \pm .0033$	$1.1840 \pm .0004$	$1.1822 \pm .0005$
5.	91.30	$1.1639 \pm .0002$	$1.1592 \pm .0009$	$1.1667 \pm .0033$	$1.1636 \pm .0005$	$1.1619 \pm .0005$
6.	91.95	$0.8738 \pm .0002$	$0.8711 \pm .0007$	$0.8856 \pm .0028$	$0.8769 \pm .0003$	$0.8742 \pm .0004$
7.	93.00	$0.4771 \pm .0002$	$0.4761 \pm .0005$	$0.4808 \pm .0019$	$0.4814 \pm .0001$	$0.4796 \pm .0002$
8.	93.70	$0.3521 \pm .0002$	$0.3512 \pm .0004$	$0.3521 \pm .0013$	$0.3556 \pm .0001$	$0.3550 \pm .0001$
	(b) BARE $acol_{max} = 25^0$					
1.	88.45	$0.4854 \pm .0003$	$0.4808 \pm .0005$	$0.4699 \pm .0016$	$0.4833 \pm .0003$	$0.4826 \pm .0002$
2.	89.45	$0.6746 \pm .0003$	$0.6699 \pm .0006$	$0.6593 \pm .0023$	$0.6727 \pm .0003$	$0.6710 \pm .0003$
3.	90.20	$0.9438 \pm .0003$	$0.9387 \pm .0008$	$0.9279 \pm .0033$	$0.9425 \pm .0003$	$0.9384 \pm .0004$
4.	91.19	$1.2198 \pm .0003$	$1.2130 \pm .0010$	$1.2169 \pm .0034$	$1.2187 \pm .0004$	$1.2133 \pm .0005$
5.	91.30	$1.1989 \pm .0003$	$1.1924 \pm .0010$	$1.1995 \pm .0034$	$1.1982 \pm .0004$	$1.1928 \pm .0005$
6.	91.95	$0.9054 \pm .0002$	$0.9011 \pm .0007$	$0.9124 \pm .0026$	$0.9089 \pm .0003$	$0.9014 \pm .0003$
7.	93.00	$0.5040 \pm .0002$	$0.5013 \pm .0005$	$0.4996 \pm .0019$	$0.5054 \pm .0002$	$0.5027 \pm .0002$
8.	93.70	$0.3777 \pm .0002$	$0.3749 \pm .0004$	$0.3689 \pm .0013$	$0.3782 \pm .0001$	$0.3771 \pm .0001$

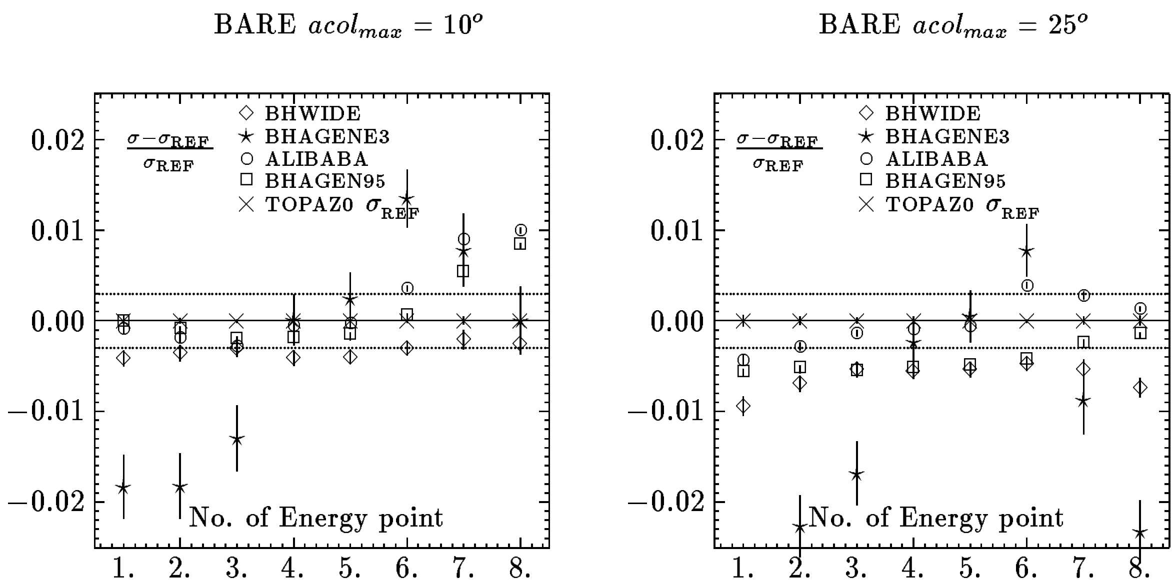

Figure 1: Monte Carlo results for BARE trigger, for two values (10^o and 25^o) of acollinearity cut. Center of mass energies (in GeV) close to Z peak. In the plots cross section $\sigma_{\rm REF}$ from TOPAZ0 is used as a reference cross section. Cross sections in nb. Two horizontal dotted lines indicate the 0.3% band, for reference.

- BARE ES – we require $40^o < \theta_{e^-} < 180^o$, $0^o < \theta_{e^+} < 180^o$, with acollinearity cuts of 10^o, 25^o and $E_{min} = 1$ GeV for both e^- and e^+;
- CALO ES – we require the same cuts as in the BARE case but with $E_{min} = 20$ GeV

No.	E_{CM}	TOPAZ0	BHWIDE	BHAGENE3	UNIBAB	BHAGEN95
	(a) CALO $acol_{max} = 10^o$					
1.	88.45	$0.4533 \pm .0004$	$0.4523 \pm .0004$	$0.4467 \pm .0008$	$0.4490 \pm .0010$	$0.4524 \pm .0001$
2.	89.45	$0.6387 \pm .0004$	$0.6377 \pm .0006$	$0.6302 \pm .0011$	$0.6358 \pm .0012$	$0.6370 \pm .0002$
3.	90.20	$0.9023 \pm .0003$	$0.9016 \pm .0008$	$0.8920 \pm .0015$	$0.9021 \pm .0014$	$0.8990 \pm .0003$
4.	91.19	$1.1725 \pm .0001$	$1.1707 \pm .0010$	$1.1767 \pm .0021$	$1.1772 \pm .0016$	$1.1689 \pm .0004$
5.	91.30	$1.1520 \pm .0001$	$1.1505 \pm .0009$	$1.1571 \pm .0020$	$1.1559 \pm .0016$	$1.1491 \pm .0004$
6.	91.95	$0.8649 \pm .0001$	$0.8646 \pm .0007$	$0.8795 \pm .0015$	$0.8689 \pm .0012$	$0.8660 \pm .0003$
7.	93.00	$0.4723 \pm .0001$	$0.4725 \pm .0005$	$0.4796 \pm .0008$	$0.4733 \pm .0008$	$0.4761 \pm .0001$
8.	93.70	$0.3486 \pm .0001$	$0.3486 \pm .0004$	$0.3507 \pm .0006$	$0.3486 \pm .0007$	$0.3526 \pm .0001$
	(b) CALO $acol_{max} = 25^o$					
1.	88.45	$0.4769 \pm .0004$	$0.4742 \pm .0004$	$0.4696 \pm .0008$	$0.4733 \pm .0010$	$0.4741 \pm .0001$
2.	89.45	$0.6638 \pm .0003$	$0.6615 \pm .0006$	$0.6556 \pm .0011$	$0.6619 \pm .0012$	$0.6599 \pm .0002$
3.	90.20	$0.9297 \pm .0003$	$0.9278 \pm .0008$	$0.9207 \pm .0012$	$0.9302 \pm .0014$	$0.9236 \pm .0003$
4.	91.19	$1.2025 \pm .0003$	$1.1994 \pm .0010$	$1.2074 \pm .0021$	$1.2073 \pm .0016$	$1.1954 \pm .0004$
5.	91.30	$1.1819 \pm .0003$	$1.1790 \pm .0010$	$1.1879 \pm .0021$	$1.1860 \pm .0016$	$1.1753 \pm .0004$
6.	91.95	$0.8924 \pm .0003$	$0.8909 \pm .0007$	$0.9058 \pm .0016$	$0.8965 \pm .0012$	$0.8899 \pm .0003$
7.	93.00	$0.4964 \pm .0003$	$0.4954 \pm .0005$	$0.5004 \pm .0009$	$0.4976 \pm .0008$	$0.4971 \pm .0001$
8.	93.70	$0.3717 \pm .0003$	$0.3704 \pm .0004$	$0.3690 \pm .0006$	$0.3720 \pm .0007$	$0.3730 \pm .0001$

CALO $acol_{max} = 10^o$ CALO $acol_{max} = 25^o$

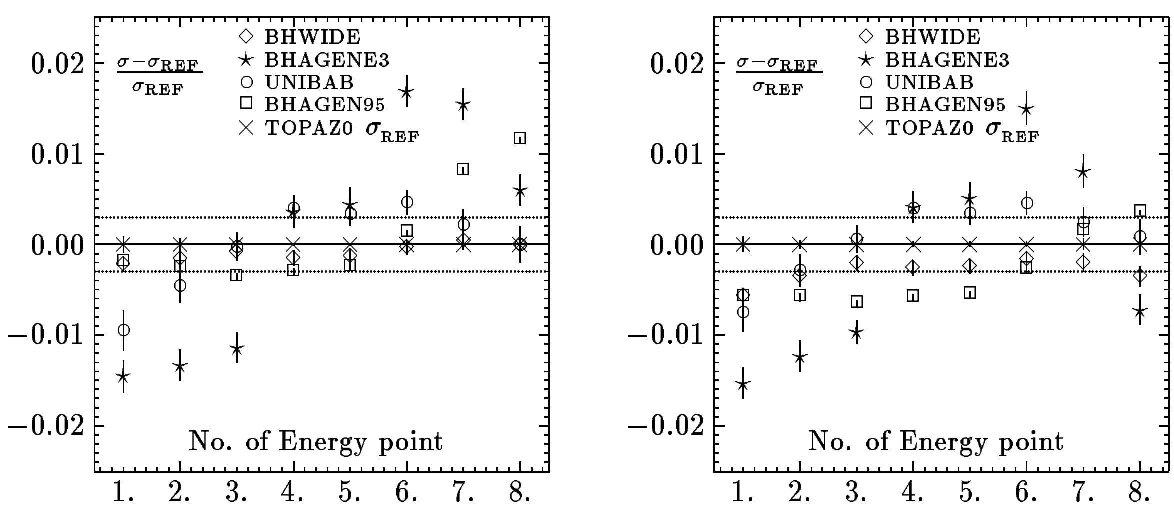

Figure 2: Monte Carlo results for CALO trigger, for two values (10^o and 25^o) of acollinearity cut. Center of mass energies (in GeV) close to Z peak. In the plots cross section σ_{REF} from TOPAZ0 is used as a reference cross section. Cross sections in nb. Two horizontal dotted lines indicate the 0.3% band, for reference.

for the final 'fermion' energy which is the $e^-(e^+)$ energy if the there are no photons nearby and which is the $e^-(e^+)$ plus the photon energy if the photon is within a cone of half-angle 1^o from the $e^-(e^+)$, respectively.

No.	BHWIDE	TOPAZ0	BHAGENE3	UNIBAB	SABSPV	BHAGEN95
	(a) CALO $acol_{max}=10^o$					
1.	$35.257 \pm .040$	$35.455 \pm .024$	$34.690 \pm .210$	$34.498 \pm .157$	$35.740 \pm .080$	$35.800 \pm .019$
2.	$29.899 \pm .034$	$30.024 \pm .020$	$28.780 \pm .170$	$29.189 \pm .134$	$30.270 \pm .070$	$30.296 \pm .016$
3.	$25.593 \pm .029$	$25.738 \pm .015$	$24.690 \pm .150$	$24.976 \pm .115$	$25.960 \pm .060$	$25.958 \pm .014$
	(b) CALO $acol_{max}=25^o$					
1.	$39.741 \pm .049$	$40.487 \pm .025$	$39.170 \pm .280$	$39.521 \pm .158$	$40.240 \pm .100$	$40.463 \pm .021$
2.	$33.698 \pm .042$	$34.336 \pm .017$	$32.400 \pm .190$	$33.512 \pm .135$	$34.100 \pm .080$	$34.287 \pm .018$
3.	$28.929 \pm .036$	$29.460 \pm .013$	$27.840 \pm .160$	$28.710 \pm .116$	$29.280 \pm .070$	$29.409 \pm .015$

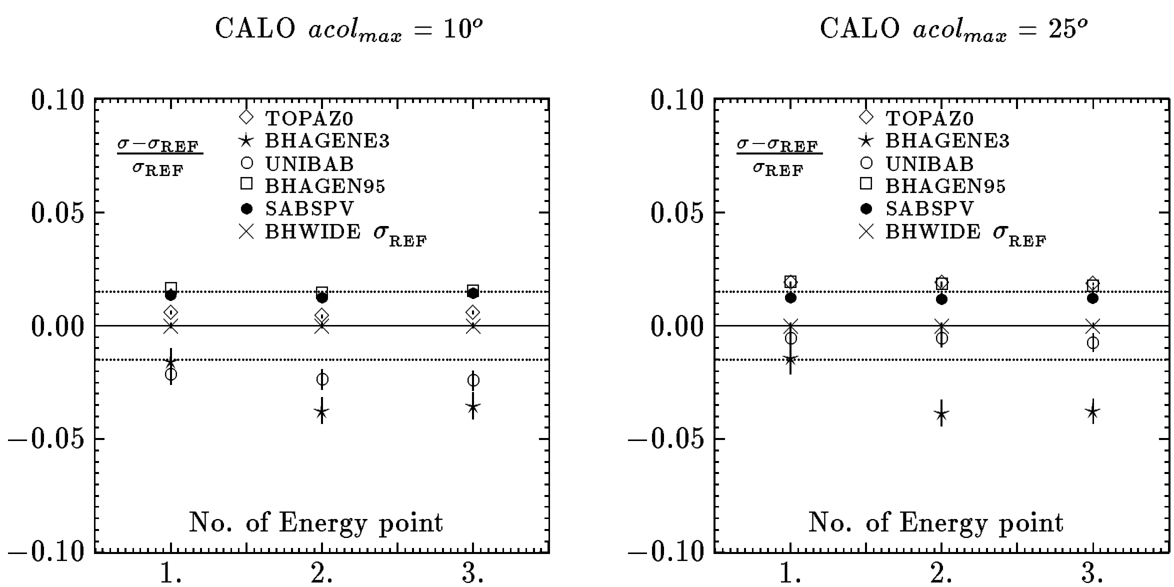

Figure 3: Monte Carlo results for CALO trigger, for two values (10^o and 25^o) of acollinearity cut. Center of mass energies close to W-pair production threshold (E_{CM}: 1. 175 GeV, 2. 190 GeV, 3. 205 GeV). In the plots cross section σ_{REF} from BHWIDE is used as a reference cross section. Cross sections in pb. Two horizontal dotted lines indicate the 1.5% band, for reference.

For the BARE ES acceptance cuts, we show in fig. 1 the comparison of the BHWIDE results with those of ALIBABA, BHAGEN95, BHAGENE3 and TOPAZ0, where for definiteness we plot the ratio $(\sigma_A - \sigma_{REF})/\sigma_{REF}$ for each calculation A, $A =$ ALIBABA, BHAGEN95[3], BHAGENE3, BHWIDE, and TOPAZ0, using the TOPAZ0 cross section as σ_{REF}. This we do for the 8 CMS energy values: 88.45, 89.45, 90.20, 91.19, 91.30, 91.95, 93.00, 93.70 GeV, which are denoted in the figure as energy points $1, 2, \ldots, 8$, respectively. We see that BHWIDE agrees with the semi-analytical programs, ALIBABA and TOPAZ0, to within a few per mille at the Z peak whereas off the peak BHWIDE remains generally within 1% of the semi-analytical programs. The difference between BHWIDE and the Monte Carlo program BHAGENE3 is also at the few per mille level at the Z peak

[3] Here we use recently updated results of BHAGEN95, obtained from the authors.

but it is as much as 2.27% off the peak. The agreement between BHWIDE and BHAGEN95 is also at the few per mille level at the Z peak; off the peak, two programs remain within 1% of one another, with the better agreement holding for the looser acollinearity cut of 25°. These results give us a handle on the total precision of BHWIDE in the Z resonance regime, as we discuss presently.

Continuing in this way, we show in fig. 2 the similar type of comparison of the predictions of the programs with the CALO ES. In fig. 2 ALIBABA is no longer applicable (it does not handle the CALO ES) and UNIBAB appears — it was too slow to participate in the BARE ES comparisons. Again, we use TOPAZ0 for the reference cross section and we plot the same ratio $(\sigma_A - \sigma_{REF})/\sigma_{REF}$ for the same 8 energy points as in fig. 1. At the peak, BHWIDE is within a few per mille of TOPAZ0 and BHAGEN95 and it is within 7 per mille of UNIBAB and BHAGENE3. Off peak, BHWIDE remains within 6 per mille of TOPAZ0 whereas it remains within 0.8% of UNIBAB, within 1.2% of BHAGEN95 and within 1.8% of BHAGENE3; the better agreement with the latter three programs holds for the looser acollinearity cut whereas for TOPAZ0 the situation is reversed. Based on these and related comparisons as described in ref. [20], including both the results and the physics approximations in the various programs as presented in this last reference, we conclude that, for the CALO ES, within ± 100 MeV of the Z peak, the total precision of BHWIDE is 3 per mille and off peak, within $+2.5/-2.75$ GeV thereof, we set this precision at 5 per mille in the LEP1 energy regime. For reference the three per mille band is indicated by the two horizontal dotted lines in figs. 1 and 2. This precision tag should be compared to that for the only other published wide angle Bhabha scattering Monte Carlo event generator in wide use at LEP/SLC, namely BABAMC [6], whose total precision on pure QED was set at 1% in the Z peak region in refs. [28, 29]. (In practice, ALIBABA and/or TOPAZ0 would be used to determine the non-QED and higher order QED part of the respective cross section.)

Turning next to LEP2 energies, we show in fig. 3 the comparison of the results of the six programs BHAGEN95, BHAGENE3, BHWIDE, SABSPV, TOPAZ0, and UNIBAB in the same format as in figs. 1, 2, using in this figure BHWIDE for the reference cross section σ_{REF} for the three CMS energy points 175, 190, and 205 GeV. We use the CALO ES only here. SASPV and BHAGEN95 are within 2% of BHWIDE for both acollinearity cuts, with SASPV within 1.5% of BHWIDE and with BHAGEN95 maximally at 0.55% above SASPV. TOPAZ0 and UNIBAB deviate from BHWIDE by as much as 1.9% and 2.4% respectively, with the worse agreement holding for the looser (tighter) acollinearity cut, respectively. These deviations are consistent with the leading logarithmic accuracy one expects for these two Z peak optimized codes. Similarly, the deviations between BHWIDE and BHAGENE3 by as much as 3.8% as shown in fig. 3 are also consistent with what one expects from the leading logarithmic Z peak optimized nature of BHAGENE3. On the basis of these and related comparisons, we estimate the total precision of BHWIDE at the LEP2 energies as 1.5%, conservatively. For reference, the 1.5% band is indicated by the two horizontal dotted lines in fig. 3.

We end this section by noting that, in addition to the comparisons just presented, we have also checked that the pure QED $\mathcal{O}(\alpha)$ predictions of BHWIDE are within 0.05% of those of the exact $\mathcal{O}(\alpha)$ MC OLDBIS of Ref. [22] — a modernized version of the MC of Ref. [30]. This gives us additional confidence in the technical component of our total precision estimate for BHWIDE 1.00.

5 Conclusions

In this paper, we have presented a new multiple photon Monte Carlo for wide angle Bhabha scattering at LEP1/SLC and LEP2 energies in which the respective multiple photon effects are realized on an event-by-event basis and in which the infrared singularities are cancelled to all orders in α via YFS exponentiation. This Monte Carlo calculation contains the exact $\mathcal{O}(\alpha)$ result and features two choices for the respective pure weak corrections at $\mathcal{O}(\alpha)$, that in ref. [1] and that in ref. [5]. It thus corresponds to the exact $\mathcal{O}(\alpha)$ YFS exponentiated treatment of wide angle Bhabha scattering.

We have illustrated our new calculation at both LEP1/SLC energies and at LEP2 energies, for two types of event selection, the BARE and CALO selections of ref. [20], which feature two choices of the acollinearity cut, 10^o and 25^o. In our illustrations, we compared our predictions with those of refs. [1, 10, 12, 14, 18, 19]. We found in general a good agreement of the various calculations, an agreement consistent with the levels of approximations and realms of applicability of the respective codes. In this way, we arrived at the precision tags of 0.3% for BHWIDE at the Z peak and of 1.5% at LEP2 energies. The program is available from the authors at the WWW URL http://enigma.phys.utk.edu/pub/BHWIDE/.

In summary, we have developed a new Monte Carlo event generator for wide angle Bhabha scattering at LEP1/SLC and LEP2 energies in which the infrared singularities are cancelled to all orders in α via YFS exponentiation of the respective multiple photon effects. We look forward with excitement to its application to LEP1/SLC and LEP2 data analyses.

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