SLD NOTE 269
July 2000

Muon-Pair Acolinearity: A Direct Measure of Beam Energy Asymmetry and a Check of the $A_{LR}(\bar{E}_{cm}) \rightarrow A^0_{LR}$ Correction

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ABSTRACT

Muon pair acolinearity is used as a direct measure of beam energy loss, an effect arising mostly from single photon emission by one beam at a time. Most of the beam energy loss is due to initial state radiation, but non-negligible contributions result from beamstrahlung and from the initial beam energy differences. The acolinearity results from beam energy asymmetry and the resultant motion of the Z^0 relative to the laboratory frame. The width of the acolinearity distribution, expressed in terms of its RMS, is directly related to the average beam energy loss. This study allows an experimental confirmation of the theoretically predicted decrease of the average center of mass energy, $\bar{E}_{cm}$, whose value is needed for transforming the measured value of $A_{LR}(\bar{E}_{cm})$ to its Z^0 pole value, i.e., $A_{LR}(\bar{E}_{cm}) \rightarrow A^0_{LR}$. From these studies, we are able to infer that the energy loss expected from the theoretical models is accurate to better than 10%. This corresponds to a $\frac{\delta A^0_{LR}}{A^0_{LR}}$ uncertainty of about 0.25%. The latter is to be compared with the total systematic uncertainty in $\frac{\delta A^0_{LR}}{A^0_{LR}}$ of 0.64% from all sources of error. We also are able to estimate from the mean value of the muon-pair acolinearity that the average electron beam energy is 28 ± 8 MeV higher than the average positron beam energy, a result that agrees with an earlier estimate for the 1997-98 SLD data set used in this analysis.

INTRODUCTION

Muon pairs from Z^0 decays, with their high energy and small energy loss, are tracked with excellent angular resolution. The muon directions are colinear when the Z^0 decays at rest. If the Z^0 is moving relative to the laboratory frame, an acolinearity of the muon pair results which increases with the momentum of the Z^0. The Z^0 motion is due to unequal beam energies resulting, in most cases, from the emission of a single hard photon traveling nearly parallel to the beam axis. For example, a 28 MeV difference between the electron and positron beam energies will result in a 28 MeV/c net momentum of the the Z^0 in the electron beam's direction. If the Z^0 decays into a muon pair whose directions are nearly perpendicular to the beam axis, there will be an acolinearity between the two muons of about 0.6 mrad.

The effective center of mass energy, or the mass of the Z^0, will be affected by the energy loss of each beam due initial state radiation and, to a much lesser extent, by disruption of one beam bunch by the other, i.e., by beamstrahlung. In addition, the mean beam energies before the bunch collisions are not exactly equal, and the energy spreads about the mean also cause a small but significant beam energy asymmetry. All three effects, initial state radiation, beamstrahlung and the characteristics of the colliding beams are simulated and compared in terms of their effect on muon pair acolinearity.

Our measurement of muon pair acolinearity is the only experimental check of the theoretically predicted sources of energy loss needed to correct electroweak parameters for energy dependence for the SLD experiment. This study shows from the data itself how initial beam characteristics, energy loss through beamstrahlung and initial state radiation affect the center of mass energy. These contributions are understood by comparing the shape and width of the observed muon pair acolinearity with simulations that incorporate these three contributing effects.

SLD's most precise measurement, the weak mixing angle, $\sin^2\theta_W^{eff}$, is determined from $A_{LR}(\bar{E}_{cm})$ after a correction for the difference between the average center of mass energy, $\bar{E}_{cm}$, and the Z^0 pole energy, M_Z. The Z^0 pole value of A_{LR}, or A^0_{LR}, is equal to A_e, from which we compute $\sin^2\theta_W^{eff}$.

To be consistent with notation used elsewhere, note that $\bar{E}_{cm}$ is the average center of mass energy after all initial photon radiation occurs. This includes photon emission from both beamstrahlung and initial state radiation. By contrast, the quantity referred to as E_{cm} in the recent PRL reporting the final A_{LR} measurement, is the average center of mass energy just before collision, i.e., after the effect of beamstrahlung, but before initial state radiation occurs. [1]

The correction to A_{LR} for hadronic final states is 1% per 80 MeV change in Z^0 energy. [2] The total energy-related correction to A_{LR} is 0.00358, or 0.00045 in $\sin^2\theta_W^{eff}$. Since the statistical error in $\sin^2\theta_W^{eff}$ is only 0.00027, it is clear that the correction based on accounting for energy loss is relatively large and important. [1]

The acolinearity is determined from the polar angles, θ^+ and θ^-, of both muons with equation 1.

$$\delta = \frac{\cos\theta_+ + \cos\theta_-}{\sin\theta_-} \tag{1}$$

This is a very good approximation, but not an exact formula. Terms of order δ^2 are negligible and ignored.

The magnitude of the correction $A_{LR}(\bar{E}_{cm}) \rightarrow A^0_{LR}$ is directly proportional to the difference, $M_Z - \bar{E}_{cm}$. The width of the distribution in δ, expressed in terms of the RMS of the distribution, has been also shown to be proportional to $M_Z - \bar{E}_{cm}$.

We begin below with a technical description of the simulation followed by a discussion of the analysis and, finally, with conclusions.

SIMULATION

Input and Initialization

- The mean energy of the electron beam is set higher than the positron beam energy by 28 MeV to match a similar shift seen in the data. In addition, each beam energy is independently generated with a gaussian spread. The sigma of the spreads compared with data were 60 MeV, 65 MeV and 75 MeV.

- The beamstrahlung energy loss per beam is read in from a file of 10,000 events generated with the program Guinea Pig.[3]

- The initial state radiation energy loss is simulated with 10,000 events generated with Unibab. [4] This includes initial state radiation. However, final state radiation is suppressed for muons. The radiated photons for each Unibab event are combined into a single 4-vector that describes the momentum and energy lost from the event. If there are no photons in the event, all of the values in the combined 4-vector are zero.

- The distribution in $\cos\theta$.of muon pairs is read in from the real data for both signs of polarization.

Event Generation

- The energy of each beam is randomly selected with a gaussian distribution whose mean and sigma is adjusted to match the data.

- To approximate the asymmetry that results from this beamstrahlung, we randomly reduce the energy of one of the two beams, not both, by reading an event from the beamstrahlung simulation file. This is tantamount to photon emission parallel to the beam axis.

- The energy loss due to beamstrahlung results in a corrected center of mass energy. The x and y components of the momentum of the simulated Z^0 are both zero since the beams are made to collide along the z-axis. Hence, the beamstrahlung energy loss effects the CM motion of the Z^0 only along the z-axis.

- An event from the initial state radiation file is randomly selected. The 4-vector describing the combined effect of all photons emitted is subtracted from the 4-vector in the previous step.
- The resultant mass of the simulated Z^0 is calculated.
- A $\mu^+\mu^-$ pair is generated in the center of mass of the simulated Z^0. The $\cos\theta$ of the μ^- is selected randomly from the data file. The ϕ angle is generated from a flat distribution.
- The $\mu^+\mu^-$ pair is boosted into the lab frame of reference.
- The acolinearity calculations are made from the $\mu^+\mu^-$ 4-vectors in the lab frame of reference.

ANALYSIS

Data Selection

The data consists of about 8000 muon pairs from the 1997-98 sample. The exact number used varies slightly with the cuts used in each substudy. Overall, the muon pair selection is the same as that used for the study of muon asymmetry parameters.

The selection criteria include: $M_{\mu\mu}$ greater than 70 GeV, the raw cluster energy must be less than 10 GeV and the cosine between tracks must be less than -0.9.

These cuts reduce the only significant background, i.e., from tau pairs, to less than 0.2%. [5]

To further enhance the quality of the selected events additional restrictions were applied. We require three VXD3 hits for each muon track. The polar angle of each muon is required to be less than 0.8 in magnitude.

In addition, we study the effect of cuts on the acolinearity projected in the x-y plane (perpendicular to the beam axis) where motion of the Z^0 is a small effect. We label this the transverse acolinearity, discussed in the following section.

Transverse Acolinearity

Since energy loss is due primarily to longitudinally emitted photons, the expected transverse Z^0 momentum is small. The Z^0 motion resulting from transverse photon emission due to initial state radiation was included in our simulations. However, the simulation of beamstrahlung energy loss did not contain a transverse component. We expect the average contribution of transverse momentum from beamstrahlung to be much smaller than that due to initial state radiation.

Accounting for the contributions to transverse Z^0 motion from both initial state radiation and from measuring resolution, we estimate a core transverse acolinearity whose mean magnitude is less than 0.5 mrad. However, the data shows a spread which is about twice this value in the central core and even larger if more of the tails in the distribution are included. This shows that the real resolution is not as good as expected, but still quite good and adequate for the study of muon pair acolinearity in the polar direction as measured by the angle δ in equation 1.

Semi-Quantitative Comparisons of Data with Simulations

Figure 1 shows a comparison between data and simulations. The data includes a cut that requires the transverse acolinearity to be less than 20 mr. This cut was chosen as a compromise. It is made tight enough to reduce background from poorly measured tracks and non-muon background. But it is not too tight a cut that would exclude too many good muon pairs that happen to have a larger transverse acolinearity.

The simulations include all three effects that result in unequal beam energies: beam smearing, beamstrahlung and initial state radiation. The central values of the two beams have been offset with the electron beam's energy 28 MeV higher than that of the positron beam. We discuss below, referring to Table 3, how we estimate the average beam offset from the mean value of the acolinearity distribution and compare our result with that estimated in an earlier report. [2]

The gaussian beam spread of each beam is fixed at $\sigma = 65$ MeV for the comparisons shown in the plots. We also made studies with $\sigma = 60$ and 75 MeV and with other beam energy offsets. The 28 MeV beam offset and the 65 MeV beam spread were chosen based on a subjective judgement of the good agreement between data and simulations, i.e., no quantitative fitting procedure was used. The agreement seen in Figure 1 between data and simulation is very good and further tuning of parameters for this application is not justified. Figure 2 is the same plot, but with the vertical scale reduced in order to emphasize the tails of the distribution. Again, agreement is seen to be quite good.

Contributions to Acolinearity (δ)

Figure 3 shows simulations of the shapes of the three contributing effects separately. Beam smearing and beamstrahlung contribute only at small values of acolinearity, or small energy loss, while initial state radiation has a very broad tail that accounts for most of the energy loss. Figure 4, like Figure 2, has a reduced vertical scale to emphasize the events in the tails of the distributions.

Note that the important thing to compare in Figures 3 and 4 are the shapes of the distributions, not the actual numbers in each bin. That is, for this comparison, the three contributing distributions don't contain the same event-by-event information as the distributions with all three contributions combined as shown in Figures 1 and 2.

Table 1 gives numerical values of the information shown in Figures 3 and 4. Again, one sees dominance of initial state radiation at large values of acolinearity, or energy loss. However, it is also important to note that one cannot ignore the contributions of beam smearing and beamstrahlung to get good agreement with the data.

Estimates of Energy Loss and Comparisons with Earlier Estimates

The average energy loss due to beamstrahlung is 37 MeV and the average energy loss due to initial state radiation is about 240 MeV, according to our simulations.

We compare the estimates from our simulations with those reported in the recent PRL and in SLD Physics Note 264, [1], [2]

The energy-related correction $A_{LR}(\bar{E}_{cm}) \rightarrow A^0_{LR}$ reported in the PRL is 0.00358 and A_{LR} is 0.149. This represents a 2.4% correction to A_{LR}. SLD note 264 indicates that an 80 MeV variation in average energy loss results in a 1% change in A_{LR} for hadronic final states. Hence the energy difference between the actual average center of mass energy and the energy at the Z^0 pole, according to these reported estimates, must be 192 MeV.

To compare our estimates of energy loss with those reported in SLD note 264 and in the PRL, we must make some additional observations. E_{cm}, as noted above, is the average center of mass energy after the beamstrahlung correction has been made. It is estimated to be 91.237 GeV. The Z^0 pole energy is 91.187 GeV. The difference between these two values is 0.050 GeV. Hence the difference $E_{cm} - \bar{E}_{cm} =$ (.192 + .050) GeV = .242 GeV or 242 MeV. This is the average energy loss due to initial state radiation. The estimate of the average beamstrahlung energy loss in SLD Note 264 is 28.4 ± 10 MeV. [2]. This is to be compared with our estimate of 37 MeV. Therefore, the energy loss due to both initial state radiation and beamstrahlung, from SLD Note 264, is (242 + 28.4) MeV = 270 MeV. Our simulation, with both initial state radiation and beamstrahlung included, gives an average energy loss of 275 MeV, in good agreement with the SLD note 264 numbers.

Quantitative Comparisons Between Data and Simulations

We use the RMS of the muon pair acolinearity distribution for a quantitative measure of energy loss. To show that this is a valid correlation, we used our simulations to study average energy loss versus RMS of the muon pair acolinearity. Events were chosen where the difference between the total initial center of mass energy and the effective Z^0 mass is determined. For values of energy loss less than 0.5 GeV, 0.5 GeV to 1.5 GeV, 1.5 GeV to 2.5 GeV and 2.5 GeV to 3.5 GeV, we determined the average energy loss within each energy interval and the RMS of the muon pair acolinearity distribution for the events in that same interval. The energy loss (RMS) values for each of the four intervals are as follows: 0.076 GeV (2.93 mrad), 0.856 GeV (15.5 mrad), 1.91 GeV (33.6 mrad) and 2.91 GeV (51.2 mrad), respectively. The RMS is seen to vary linearly with average energy loss as expected.

A quantitative comparison between data and the full simulation with all contributions included is determined by computing the RMS of the acolinearity distributions for different maximum values of the acolinearity, δ. The maximum acolinearity was taken to be 20, 40, 80 and 160 mrad. Table 2 gives RMS values with errors are compared with simulations.

Table 3 shows the mean values of the acolinearity derived from event distributions of the variable δ as defined in equation 1. The mean values are calculated from simulations where the difference between the central values of the electron beam energy and the positron beam energy is varied. The difference $E_{electron} - E_{positron}$ is taken to be 20, 28 and 36 MeV. In addition, the mean values of the acolinearity are studied with different sized limits for the maximum values of δ. The maximum acolinearity limits chosen are 40, 80 and 160 mrad.

The comparisons between simulations and data include errors estimates for the data which have been determined from an ensemble of data-sized sets of simulations. We have also examined the effect of the transverse acolinearity cut, showing values for 20 mrad and 50 mrad. Two separate beam smearing widths are shown for the simulations, 65 and 75 MeV.

Overall, the agreement is quite good. The 75 MeV beam spread gives values closer to the data when the transverse acolinearity cut is chosen to be 20 mrad. However, we found that the shapes of the distribution, especially near the peak of the acolinearity distribution, were better reproduced with a $\sigma = 65$ MeV while also including the effect of beamstrahlung.

We estimate the average $E_{electron} - E_{positron}$ difference to be 28 ± 8 MeV. Our estimate for the central value and error is based not only on the numbers in Table 3, but also on the semi-quantitative comparison of data with simulations as shown in Figures 1 and 2. An earlier estimate of this offset based on muon pair acolinearity reported was reported to be 34 ± 8 MeV. [2]

CONCLUSIONS

We have shown that substantial agreement is obtained in comparing the distributions, data versus simulations, of the muon pair acolinearity as defined by equation 1. The simulations include three effects: beam smearing, beamstrahlung and initial state radiation. Most of the effect is due to initial state radiation which exhibits a large tail in the acolinearity distribution. Beam energy smearing, due to the intrinsic properties of the beams, and beamstrahlung, due to beam-beam disruption, result on-average to smaller energy losses that contribute mostly to the central peak of the muon pair acolinearity distribution.

We also compare results with an independent study of the mean muon pair acolinearity done earier for the purpose of estimating the difference between the average energies of the electron and positron beams. [6] That study reported the average electron-positron beam energy difference to be 34 ± 8 MeV, with the electron beam energy higher than that of the positron beam. Our estimate is 28 ± 8 MeV. The small difference between the two estimates is likely due to different criteria in event selection between the earlier study and ours.

The widths of the acolinearity distributions are measured in terms of the RMS of the distributions. The width of the acolinearity distribution is a measure of the energy asymmetry between the two beams as a result of beam energy loss, or, equivalently, to the variation of the effective Z^0 mass. From the comparisons in Table 2, we judge that the agreement between RMS widths of acolinearity distributions, data versus simulations, for a wide variety of event selection criteria, is better than about 10%. In most comparisions shown in Table 2, the agreement is much better than 10%, but this level of agreement is good enough and pressing for a more accurate smaller figure is not essential for our purposes.

Our 10% estimate on the level of agreement between data and simulations infers that the energy variation on which the $A_{LR}(\bar{E}_{cm}) \rightarrow A^0_{LR}$ correction is based is also correct to better than 10%, or about 0.00036. Since A^0_{LR} is about 0.15, this represents an uncertainty of about 0.25% in $\frac{\delta A^0_{LR}}{A^0_{LR}}$ which is to be compared with a total uncertainty of 0.64% in $\frac{\delta A^0_{LR}}{A^0_{LR}}$. [1]

Finally, we should note how this analysis can be done more rigorously. The improvements that could be made in the study could include a more accurate simulation of beamstrahlung that incorporates all of the information about emitted photons and their directions in three dimensions on an event-by-event basis. The simulation of initial state radiation was done in this kind of detail, and it is by far the dominant effect, so our approximate simulation of beamstrahlung should be adequate. Also, a better understanding of the measuring resolution could account for the transverse acolinearity in more detail, although we believe that improvement would have little effect on the polar angle acolinearity, which is the important variable. The simulation of the beam energy variations could, in principle, also be improved. However, it is not likely that any of these improvements would change the key conclusions mentioned above, but it would substantially delay the issuance of this note.

References

[1] *A High-Precision Measurement of the Left-Right Z Boson Cross-Section Asymmetry*, Phys.Rev.Lett.84:5945-5949,2000.

[2] *Calibration of the WISRD Energy Spectrometer with a Z Peak Scan* by P.C. Rowson, R. Frey, S. Hertzbach, R. Kofler, M.Swartz, M. Woods, SLD Note 264, July 27, 1999; Also, private communication with P. Rowson.

[3] *Results of Guinea-Pig Simulation for Energy Loss due to Beamstrahlung for 1997-98 SLD Run*, by M. Woods, SLD Note 263, June 1999; Also, private communication with M. Woods.

[4] UNIBAB is a program that simulates wide angle Bhabha scattering for polarized beams. *UNIBAB, Version 2.0: Monte Carlo Event Generation for Large-Angle Bhabha Scattering at LEP*, by H. Anlauf, H.D. Dahmen, P. Manakos, T. Mannel, H. Meinhard and T. Ohl, CERN-TH 7056/93, October 1993 and Fortran Code, September 30, 1994.

[5] See paper submitted to PRL titled *An Improved Direct Measurement of Leptonic Coupling Asymmetries with Polarized Z Bosons*, June 2000.

[6] The earlier estimate of a non-zero acolinearity as a reflection of the electron-positron beam energy difference was made by using data provided by Su Dong, as it applied for the 1997-98 data. The report of this earlier estimate is reported in SLD Note 264. [2]

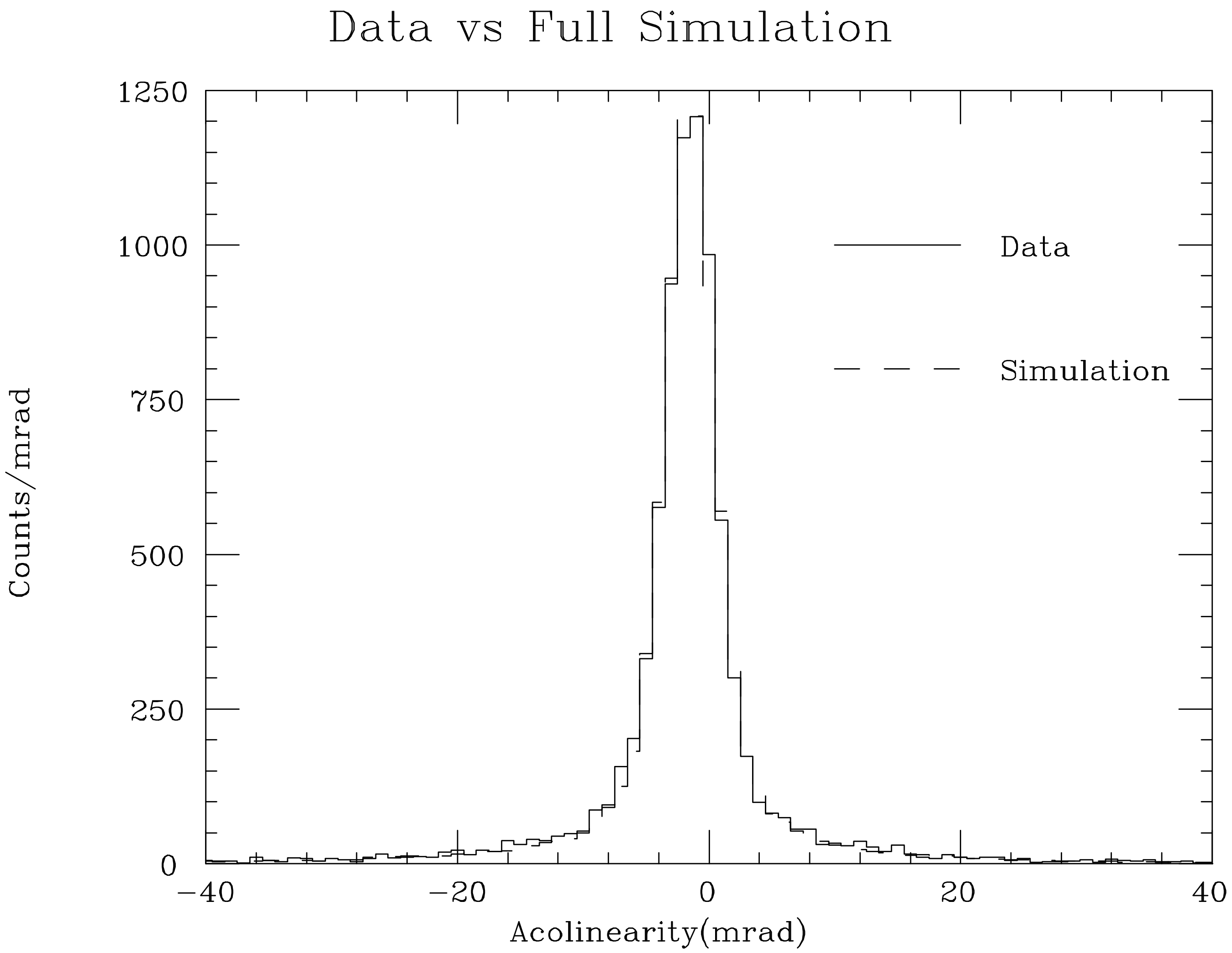

Figure 1: Measured acolinearity compared with the full simulation which includes beam smearing, beamstrahlung and initial state radiation.

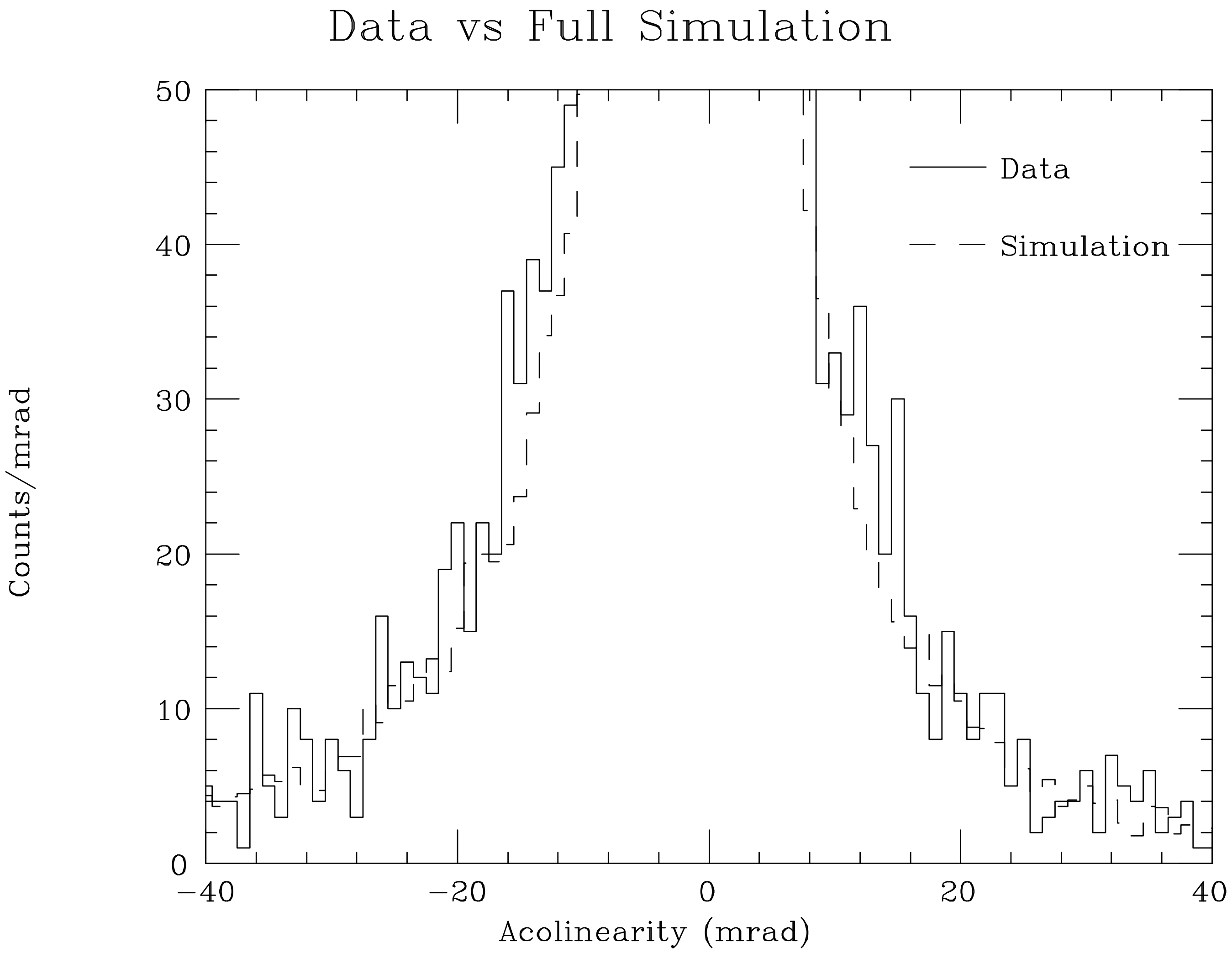

Figure 2: Measured acolinearity compared with the full simulation which includes beam smearing, beamstrahlung and initial state radiation. Same as Figure1, but with vertical scale changed to emphasize tails of distribution.

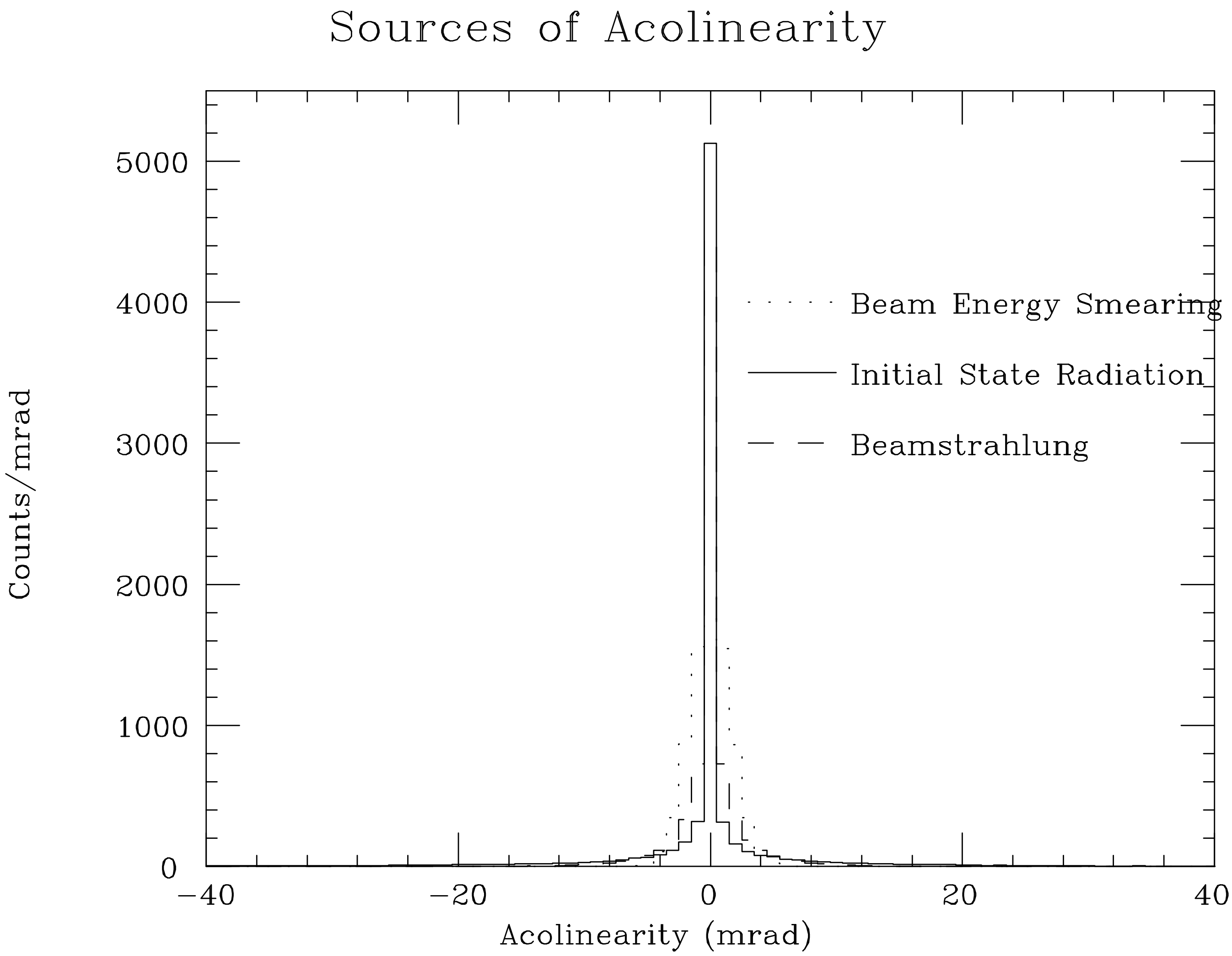

Figure 3: Simulations showing contributions to acolinearity from beam smearing, beamstrahlung and initial state radiation.

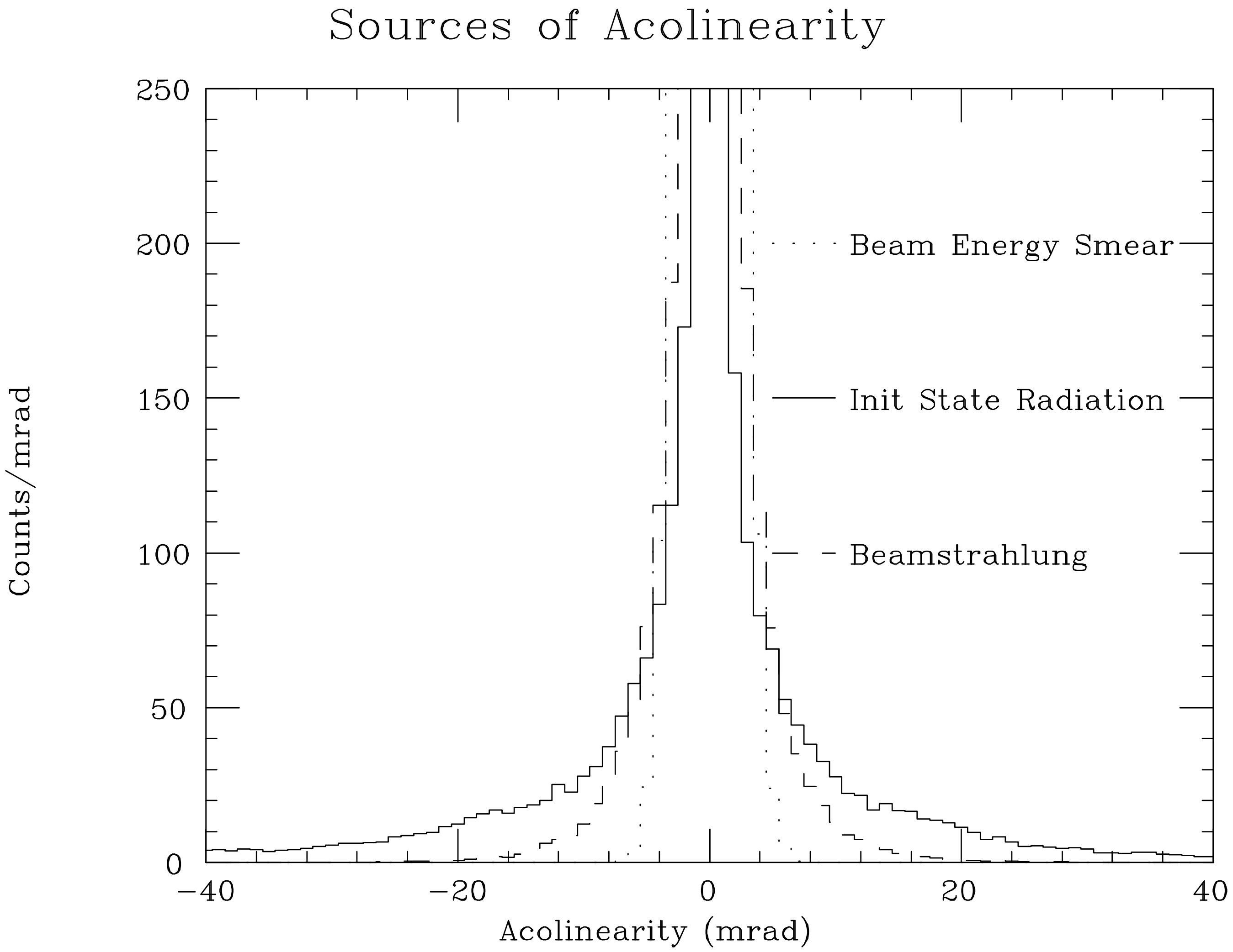

Figure 4: Simulations showing contributions to acolinearity from beam smearing, beamstrahlung and initial state radiation. This figure, like Figure 3, emphasizes the tails of the distributions.

Table 1: Sources of Acolinearity

Acolinearity (mrad)	Beam Smear	Init. State Rad.	Beamstrahlung
-20		12.5	.7
-19		14.5	1.1
-18		15.7	1.5
-17		16.9	1.9
-16		15.9	1.8
-15		17.8	2.8
-14		18.6	4.1
-13		20.0	6.3
-12		25.2	7.4
-11		22.8	8.8
-10		27.9	12.4
-9		31.1	19.0
-8		37.4	25.4
-7	0.4	47.3	36.0
-6	4.4	57.9	50.2
-5	24.3	66.0	76.1
-4	104.0	83.4	115.5
-3	347.2	115.3	187.5
-2	867.4	173.0	332.3
-1	1560.0	317.4	729.1
0	1901.2	5128.3	4456.8
1	1545.7	315.9	727.
2	862.4	158.1	333.3
3	346.8	103.5	185.3
4	106.1	79.6	115.7
5	23.9	69.0	75.8
6	4.7	52.7	48.2
7	0.5	44.4	35.2
8	.	38.2	24.6
9		32.7	18.5
10		27.7	13.1
11		22.4	9.0
12		21.7	7.4
13		17.0	6.2
14		19.1	4.1
15		16.8	2.9
16		16.6	2.2
17		14.0	1.9
18		13.6	1.4
19		12.8	1.1
20		11.4	.7

Table 2: Acolinearity RMS (mrad)

Acol_{MAX} (mrad)	Data/20 mr	Data/50 mr	Sim/65 MeV	Sim/75 MeV
20	4.85 ±.07	4.93 ±.07	4.57	4.89
40	7.02 ±.17	7.37 ±.17	6.69	6.92
80	9.44 ±.20	10.61 ±.20	8.99	9.20
160	11.44 ±.48	12.89 ±.48	11.4	11.6

Table 3: Mean Acolinearity (mrad)

$E_{el} - E_{pos}$	$Acol_{max} = 40$ mr	$Acol_{max} = 80$ mr	$Acol_{max} = 160$ mr
20	-.53	-.68	-.61
28	-.68	-.78	-.77
36	-.82	-.93	-.92
Data	-.75 ± .07	-.66 ± .08	-.75 ± .11