

[Some] Applications of QCD

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Lepton and Photon Interactions at High Energies,
Stanford University, August 9-14, 1999**

Outline

I. Rules of the game

II. Perturbative Methods

- higher orders from asymptotic expansions
- towards NNLO jet calculations

III. Beyond perturbation theory : power corrections

- event shapes
- the plaquette

IV. Perturbative resummations

- partonic thresholds
- non-relativistic quarks
- small- x

V. Novel factorization theorems

- hard diffractive scattering
- skewed parton distributions
- exclusive B decays

VI. Summary of a summary

Factorization

Take quarks & gluons seriously

↪ hard, collinear, soft,

$$\hat{\sigma} \sim \hat{\sigma}(\mu) \otimes F(\mu)$$

"hard", short-distance,
perturbative

1 parameter α_s
[+ quark masses]

"ultraviolet
parameters"

perturbative
calculations

long-distance,
"infrared parameters"

(vacuum condensates,
parton distributions,
fragmentation functions
....)

lattice

experiment

models

Higher Order Calculations

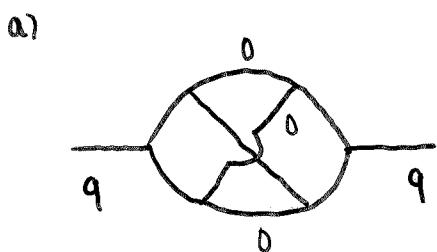
- Higher Orders from asymptotic expansions
- The new frontier :
NNLO (next-to-next-to-leading order)
jet calculations

Higher Orders from asymptotic expansions

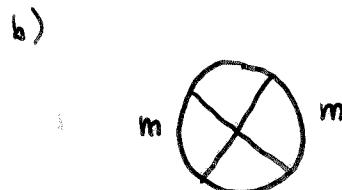
- 4-loop β -function
- 4-loop mass anomalous dimension
- α_s^3 to $e^+e^- \rightarrow$ hadrons, DIS sum rules

van Ritbergen, Vermaseren,
Larin

Chetyrkin; Vermaseren,
Larin, van Ritbergen



3 loop, massless, propagator



3 loop, massive, tadpole

Quantities with multiple scales (m, q - say) or
on-shell lines require new methods :

asymptotic expansion in m^2/q^2 , q^2/m^2 or
around other special kinematic limits

+

(approximate or exact) summation

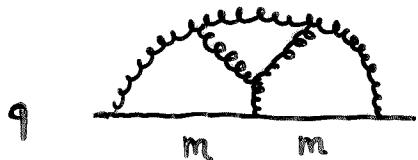
has turned out to be very useful

Analytic problems are traded in for algebraically
extensive tasks done by computers

Higher Orders from asymptotic expansions

Example: 3-loop pole mass (of a heavy quark)

$$m_{\text{pole}} = m_{\bar{q}\bar{s}}(m_{\bar{q}\bar{s}}) \left[1 + r_1 d_s + r_2 d_s^2 + r_3 d_s^3 + \dots \right]$$



Need:

$$q^2 = m^2$$

Chetyrkin,
Steinhauser

Expand around $q^2 = 0$ to order $(q^2/m^2)^{14} +$

Padé summation at $q^2/m^2 = 1$:

$$r_3 = 3.10 \pm 0.06 \quad [n_f = 4]$$

also:

$$e^+ e^- \rightarrow Q \bar{Q} X \quad d_s^2 \quad \text{Chetyrkin, Kühn, Steinhauser}$$

$$b \rightarrow c l \nu \quad (m_{l\nu}^2 = 0, m_c^2, (m_b - m_c)^2) \quad d_s^2 \quad \text{Gronewold, Helrich, Kniehl}$$

$$t \rightarrow b W \quad d_s^2 \quad \text{Chetyrkin, Golmankha, Steinhauser et al.}$$

$$b \rightarrow u l \nu \quad d_s^2 \quad \text{van Ritbergen (Staub, v.R.)}$$



algebraic expansion to all orders + exact
summation \rightarrow analytic result!

More applications?

e.g. 2 \rightarrow 2 scattering?

$s/t, t/s$ - expansion

⋮

Towards NNLO jet calculations

Many new NLO results

$e^+e^- \rightarrow 4 \text{ jets}$

$e^+e^- \rightarrow 3 \text{ jets with heavy quarks}$

[\rightarrow running of m_b - LEP+SLD]

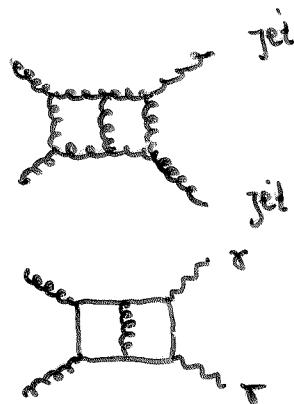
$p\bar{p} \rightarrow 3 \text{ jets}$ (soon?)

Dixon, Signer, Nagy, Tocino; Campbell, Giller, Glens; Wanke, Kauer
Bernreuther, Brandenburg, Uwer; Rodriguez, Santamaria, Sibiryak, Nelson, Okui; Tocino; Gieck, Kilgore
(only $gg \rightarrow gg$)
Kilgore

The new frontier: NNLO jets

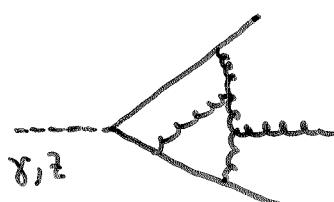
(so far only Drell-Yan $\gamma\gamma \rightarrow 2 \rightarrow 1$ scattering)

$p\bar{p} \rightarrow 2 \text{ jets}$ (1 jet inclusive)



$p\bar{p} \rightarrow \gamma\gamma X$ (Higgs background)

$e^+e^- \rightarrow 3 \text{ jets}$



↪ a_s

↪ jet structure

↪ confidence that QCD works

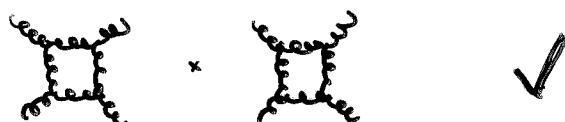
Towards NNLO jet calculations

Components

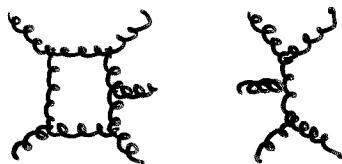
2-loop virtual \times tree



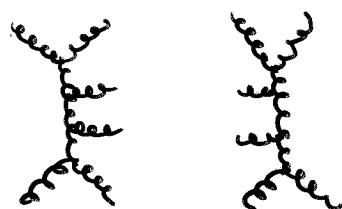
$[1\text{-loop virtual}]^2$



1-loop virtual \times tree



$[\text{tree}]^2$

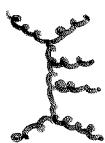


- Amplitudes to necessary accuracy [N_{LO}]
- Analytic singular phase space + numerical integration over non-singular phase space

Towards NNLO jet calculations

Progress has been made on all components

[tree]²

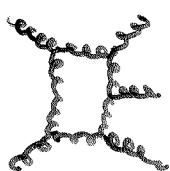


soft + collinear limits

$q_1 \rightarrow q_2 q_3 q_4$ splitting functions
have been obtained

Campbell, Glover,
Cahai, Gravitini

1-Loop virtual \times tree

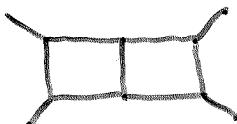


need $O(\epsilon^2)$ terms in singular phase space regions

use universal properties in soft/collinear limits to obtain the amplitude to all orders in ϵ

Bern, Del Duca,
Schmidt
+ Kilgore;
Kosower, Thor

Recent breakthrough: 2 - loop virtual

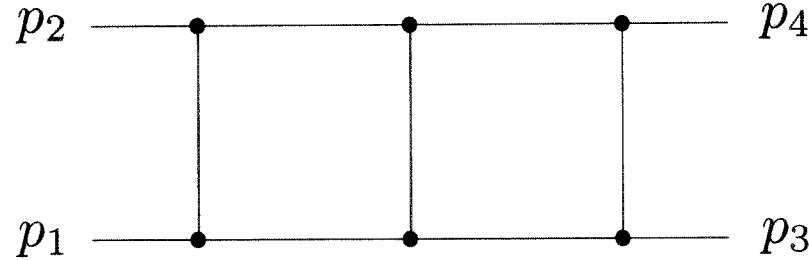


analytic result for
planar, on-shell, massless
double box with any
numerator

Smirnov;
Smirnov,
Vertutui

Method : d - parameters + Mellin- Barnes
+ summation
(was unexpected that this works !)

The planar double box



$$= \frac{(ie^{-\gamma_E \epsilon})^2}{(-s)^{2+2\epsilon}(-t)} K_1(t/s, \epsilon)$$

$$\begin{aligned}
K_1(x, \epsilon) &= -\frac{4}{\epsilon^4} + \frac{5 \ln x}{\epsilon^3} - \left(2 \ln^2 x - \frac{5}{2} \pi^2 \right) \frac{1}{\epsilon^2} \\
&- \left(\frac{2}{3} \ln^3 x + \frac{11}{2} \pi^2 \ln x - \frac{65}{3} \zeta(3) \right) \frac{1}{\epsilon} \\
&- \left[2 \text{Li}_3(-x) - 2 \ln x \text{Li}_2(-x) - (\ln^2 x + \pi^2) \ln(1+x) \right] \frac{2}{\epsilon} \\
&+ \frac{4}{3} \ln^4 x + 6 \pi^2 \ln^2 x - \frac{88}{3} \zeta(3) \ln x + \frac{29}{30} \pi^4 \\
&- 4 (S_{2,2}(-x) - \ln x S_{1,2}(-x)) + 44 \text{Li}_4(-x) \\
&- 4 (\ln(1+x) + 6 \ln x) \text{Li}_3(-x) \\
&+ 2 \left(\ln^2 x + 2 \ln x \ln(1+x) + \frac{10}{3} \pi^2 \right) \text{Li}_2(-x) \\
&+ (\ln^2 x + \pi^2) \ln^2(1+x) \\
&- \frac{2}{3} (4 \ln^3 x + 5 \pi^2 \ln x - 6 \zeta(3)) \ln(1+x) .
\end{aligned}$$

Smirnov; Smirnov, Veretin

Towards NNLO jet calculations

The bottle neck is cracked
[assume non-planar boxes ok;
not yet for $e^+e^- \rightarrow 3$ jet - need 1 loop off-shell]



Many algebraic + numerical tasks to be done

- numerator algebra

In principle ✓
In practice ?

Bern, Kosower, Yen
(N=4 SMFT)

- stable + fast integration

In principle need NNLO DGLAP splitting functions.

In practice ?

large evolution \rightarrow large $Q^2 \rightarrow$ large x

\rightarrow constraints from first few NNLO moments

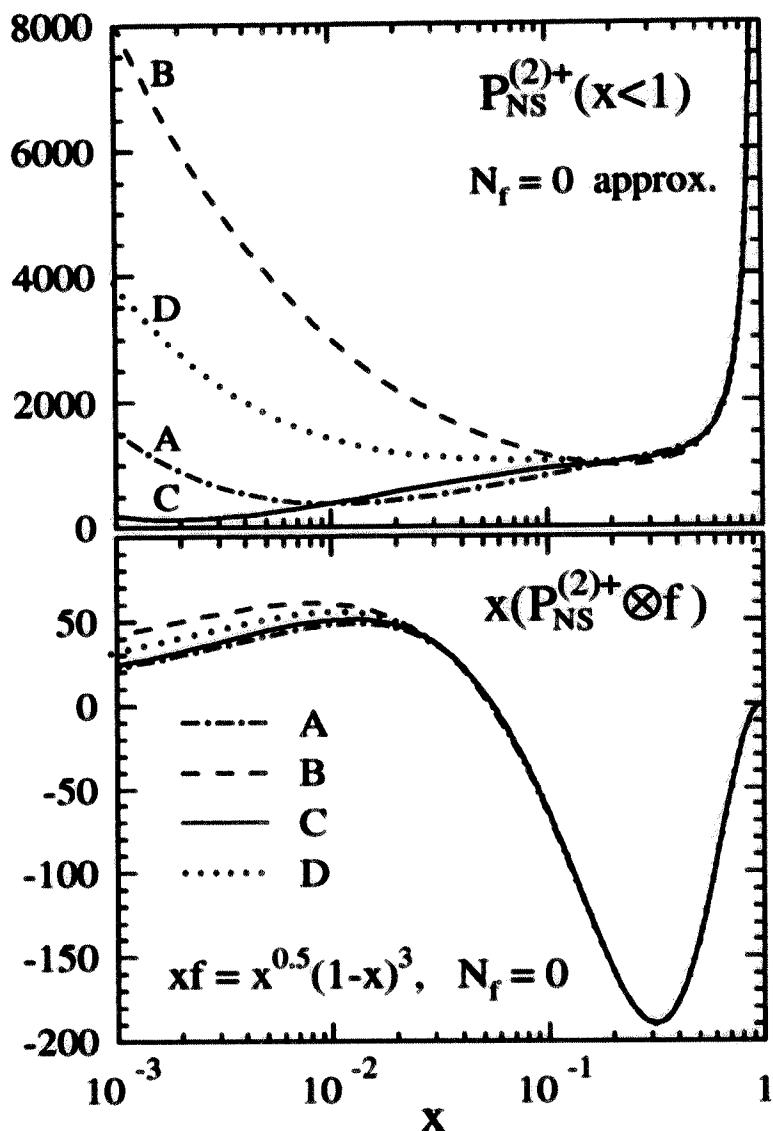
Larin, Nagurka,
van Ritbergen,
Vermaseren

- First constructions of approximate NNLO splitting functions.

Good for $x \gtrsim 0.05$

van Neerven,
Vogt

Approximate NNLO splitting functions



van Neerven, Vogt

So far non-singlet only.

Beyond Perturbation Theory

- Power Corrections
- Event Shape variables in e^+e^- - annihilation (and DIS)
- Unexpected $1/Q^2$ corrections

Power Corrections

Perturbative expansions ultimately diverge:

$$R \sim \sum_{n \gg 1} r_n d_s(Q)^{n+1}$$

$$\hookrightarrow \propto \left(\frac{2\beta_0}{\alpha}\right)^n n! n^b \quad (*)$$

In particular: divergence due to integration over small loop momenta ("IR renormalons")

perturbative long-distance sensitivity



size of perturbative coefficient



size of non-perturbative power

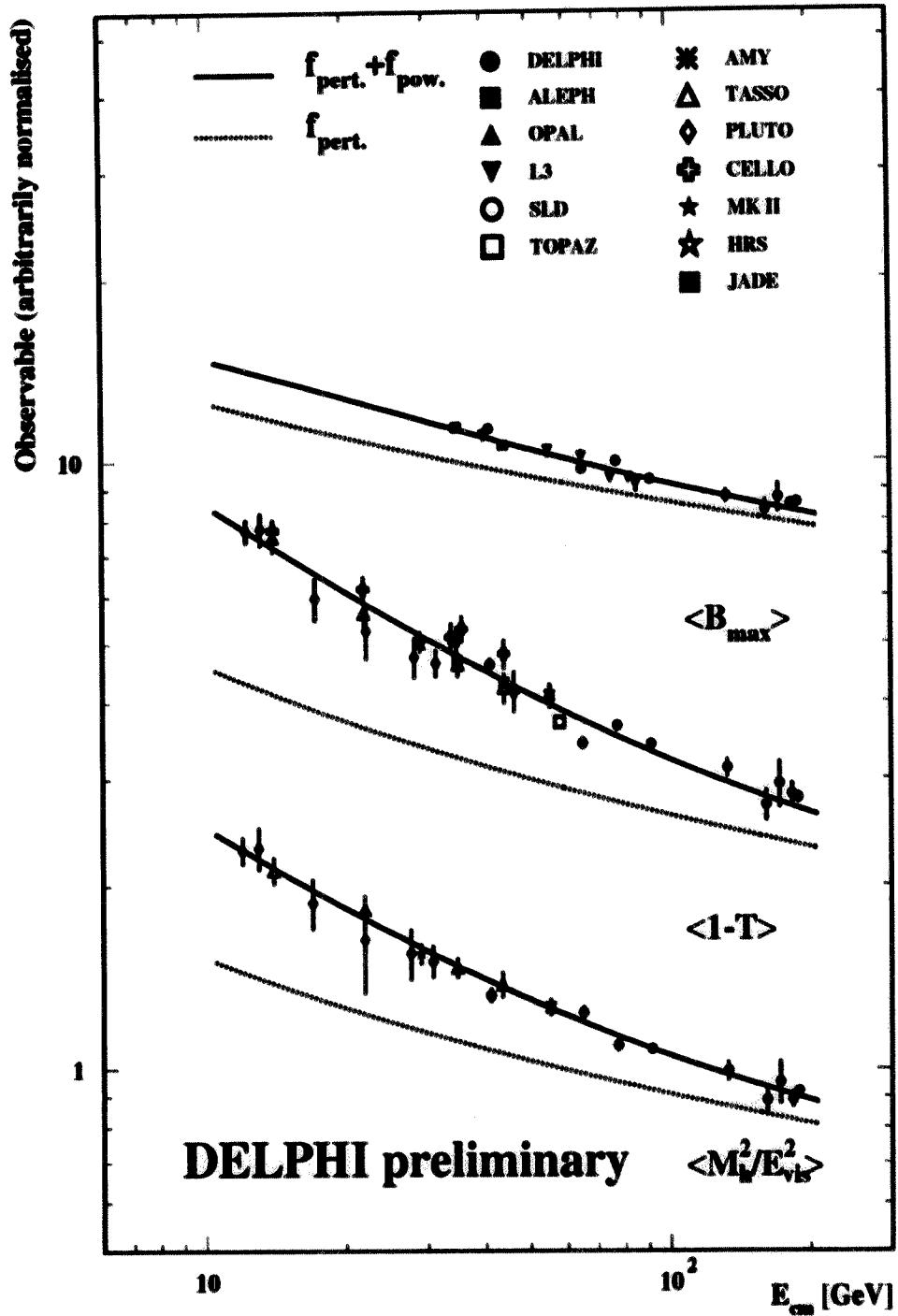
corrections

$$\left[r_n d_s(Q)^{n+1} \Big|_{\min} \underset{(*)}{\approx} e^{-\alpha/2d_s(Q)} \sim \left(\frac{\Lambda_{\text{QCD}}}{Q}\right)^\alpha \right]$$

DIS : $\alpha = 2, \dots$ "higher-twist" corrections

e+e- event shapes
fragmentation : $\alpha = 1$

Manohar, Wilen;
Weinberg, Dolgitalov, Wilen;
Altchouky, Zeldovich



Power Corrections

Event Shape : $1-T$ ($T = \text{thrust}$) , $M_{H_1}^2/\alpha_s^2$ (jet masses)
 S $\text{EEC}(x)$, $B_{W,T}$ (jet broadening)

$$\langle S \rangle = \underbrace{\langle S^{\text{pert.}}_{(\mu_I)} \rangle}_{\text{average}} + \frac{\mu_I}{Q} \langle S^{\text{NP}}_{(\mu_I)} \rangle + \mathcal{O}\left(\frac{1}{Q^2}\right)$$

- fits data well (average + distributions)

Common parametrisation

$$\langle S \rangle = A_S \alpha_S(m) + \left(B_S - A_S \beta_0 \ln \frac{\mu^2}{Q^2} \right) \alpha_S(m)^2$$

Dokshitzer, Lubatti,
D'Emilio + Nachtmann:

$$+ \frac{4 C_F}{\pi^2} \cdot 1.8 \cdot c_S \frac{\mu_I}{Q} \left\{ \bar{\alpha}(m_I) - \text{pert. subtraction} \right\}$$

2-loop correction factor

Dokshitzer, Lubatti, Nachtmann,
Salam

+ complications for jet broadenings Dokshitzer, Nachtmann, Salam

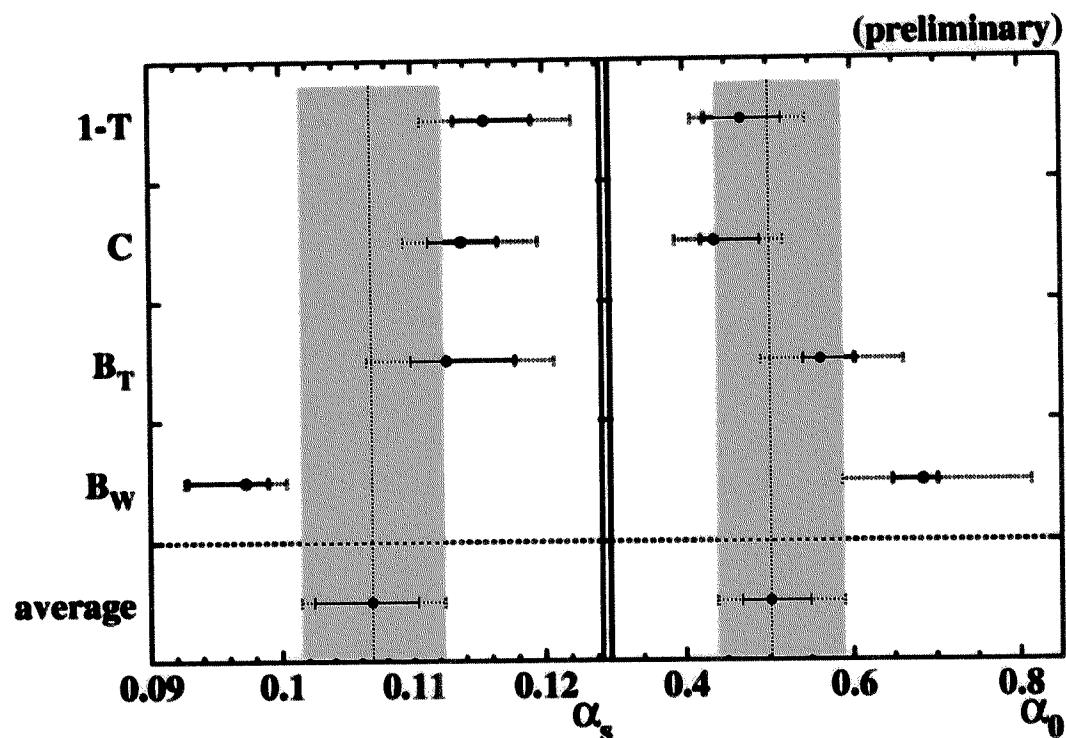
■ $\bar{\alpha}(m_I)$ argued to be universal (S -independent)

- epe-data [DELPHI, L3, OPAL, JADE]

- DIS [F2, E615]

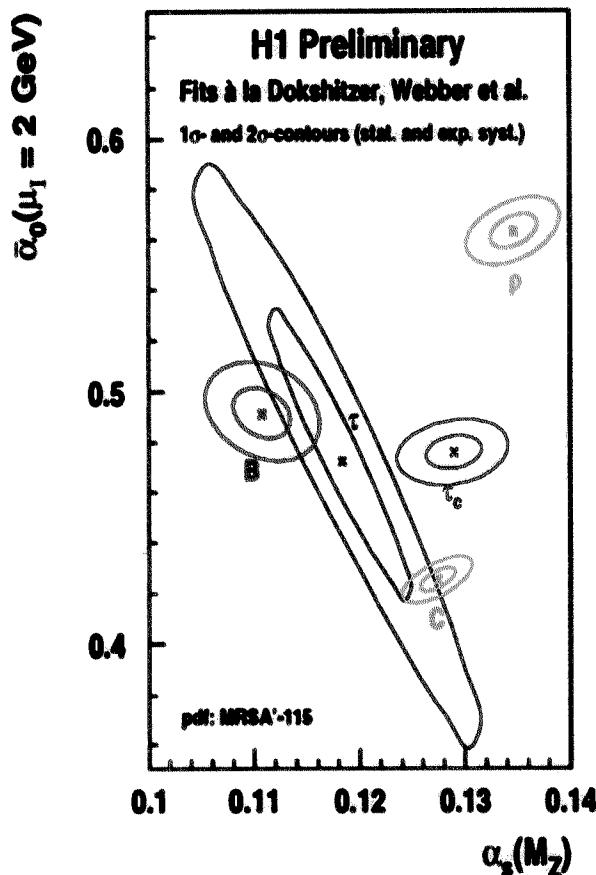
■ Universality is not a prediction of QCD

From event shape distributions in e^+e^-



Movilla-Fernandez, Biebel, Bethke

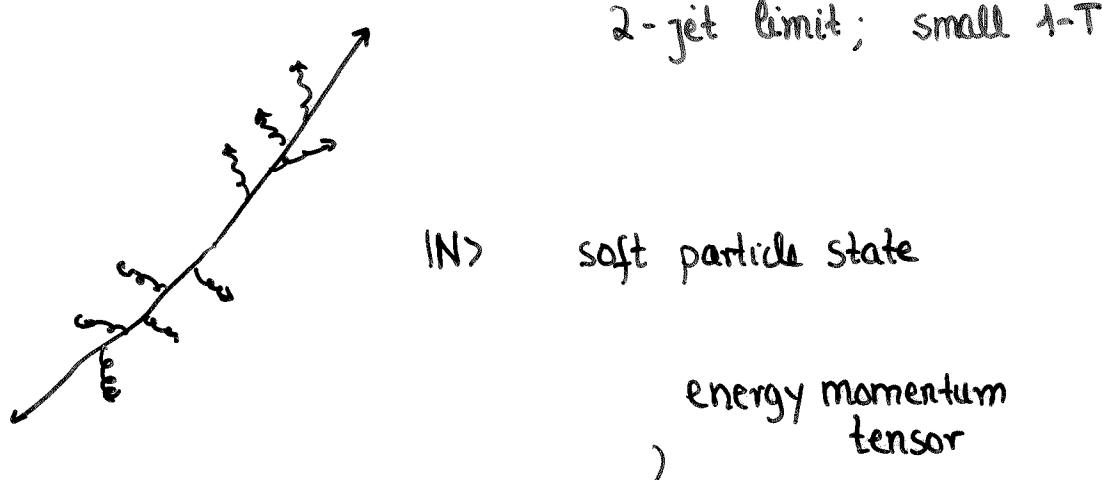
From event shape variables in DIS



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Power Corrections

- need: factorisation theorem for soft gluons beyond leading power



$$\begin{aligned} \mathcal{E}(\vec{n})|N\rangle &= \lim_{|\vec{r}| \rightarrow \infty} \int_0^\infty \frac{dr_0}{(2\pi)^2} |\vec{r}|^2 \hat{T}_i \theta_{0i}(r) |N\rangle \\ &= \sum_{i=1}^N \delta(\cos\theta - \cos\theta_i) \delta(t - r_i) E_i |N\rangle \end{aligned}$$

measures total energy flow of soft particles

The universal, non-perturbative objects are:

$$G(\vec{n}_1, \dots, \vec{n}_k; \mu_I) = \langle 0 | W^+ \prod_{i=1}^k \mathcal{E}(\vec{n}_i) W | 0 \rangle$$

!

Korchemsky, Odorico, Schrempp
Korchemsky, Schrempp
(Smirnov, Tkachov)

Important for factorisation:
 α - independent

Power Corrections

■ Event Shape distributions

$$\frac{d\sigma}{ds} = Q f_s(s_Q) R_s^{\text{PT}}(0) + \int_0^{s_Q} d\epsilon f_s(\epsilon) \frac{d\sigma^{\text{PT}}(s-\epsilon/Q)}{ds}$$

"Shape function": observable-dependent
Q-independent

$$\int d\epsilon \epsilon^k f_s(\epsilon) = \int \left[\prod_{i=1}^k dn_i w_s(n_i) \right] G(n_1, \dots, n_k)$$

■ Averages

Korchemsky,
Sterman

$$\langle EEC(X) \rangle_{1/Q} = \frac{1}{Q} G(n(x))$$

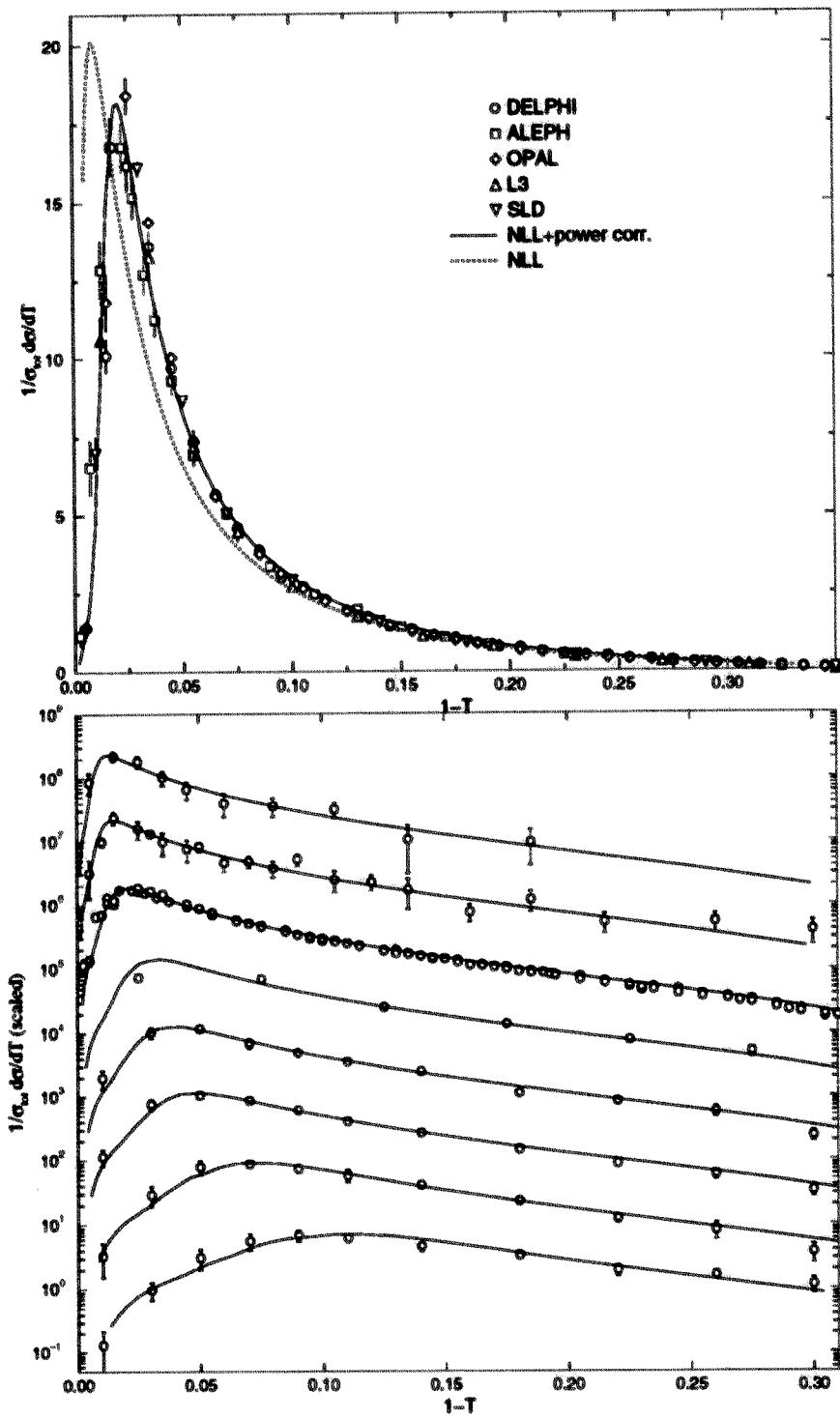
$$\langle S \rangle_{1/Q} = \int d\vec{n} w_s(\vec{n}) G(\vec{n})$$

↑
calculable weight

→ angular dependence of $G(\vec{n})$ is needed for relations between averages

Very important step towards understanding soft power corrections in QCD.

Concept of energy flow deserves more attention!



Korchemsky; Korchemsky, Sterman

1-T distributions:

Upper figure: $Q = 91.2$

Lower figure: $Q = 14, 22, 35, 44, 55, 91, 133, 161$ GeV

Power Corrections

Operator product expansion (OPE) on the lattice

$$\langle \square \rangle = \sum_{n=1}^{\infty} C_n a_s^{latt}(a) + C_{(a_s^{latt})} \cdot \frac{1}{Q^4} \langle \frac{g_s}{\pi} G G \rangle_{latt} + \dots$$

↑
plaqette
expectation
value

↑
computed to 8th (!)
order [di Renzo et al.],
summed and
subtracted

$$Q = 1/a$$

(a lattice
spacing)

Expect: $1/Q^4$ scaling

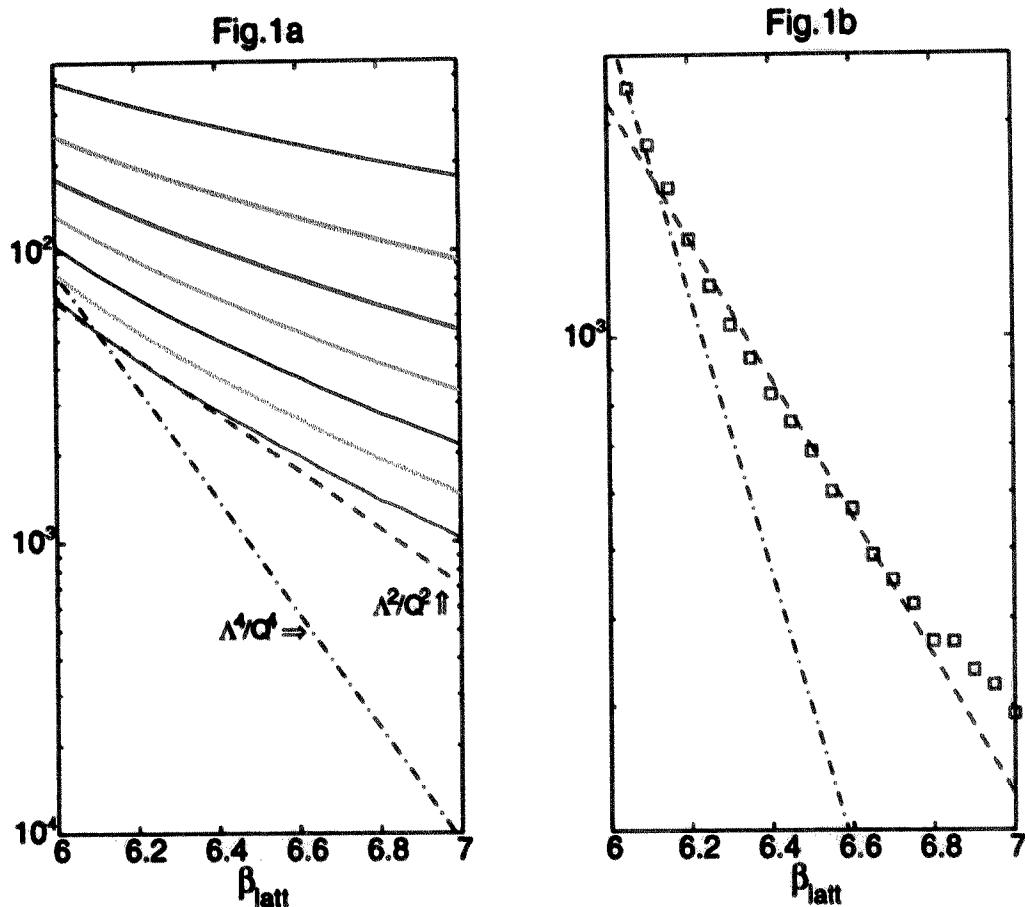
Burkard, di Renzo,
Marchesini, Guagnelli

Find: $1/Q^2$ scaling

Lattice artefact or a new type
of non-OPE power correction?

This should be cleared up!

Operator product expansion of the plaquette expectation value



Burgio, di Renzo, Marchesini, Onofri

$1/Q^2$ behaviour after perturbative subtraction!

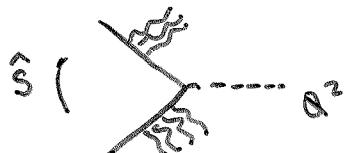
Perturbative Resummations

- Partonic Thresholds
 $x \rightarrow 1, \dots$
- Non-relativistic heavy
quarks $v \rightarrow 0$
- Small- x , high-energy
behaviour

Partonic Thresholds

$$d\sigma \sim \sum_{i,j} f_i/A \otimes f_j/B \otimes \hat{d\sigma}_{i+j \rightarrow f}$$

$$\sum_n d_S^n \frac{\ln^{2n-1}(1-z)}{[1-z]_+} + \text{subleading}$$



$$z = \frac{Q^2}{\hat{s}} \rightarrow 1$$

kinematic constraint for real parton emission

Exponentiation + Resummation

Well-known to NLL for

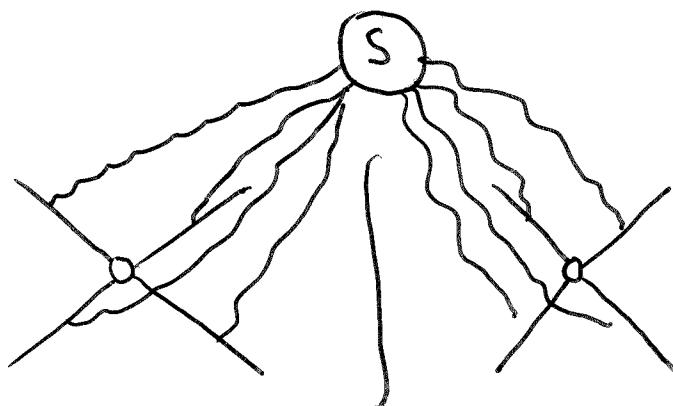
- Drell-Yan ($\mu^+\mu^-$, W, ...)
production (2 → 1)
 - Event shapes in e^+e^-
(1 → 2)

Sternen
Gebet. Traktat.

Cathartes, Tyrannidae,
Turnix, Vireos

Partoncc Thresholds

New : $2 \rightarrow 2$ scattering processes at NLL



Kidonakis, Sterman ;
Kidonakis, Odorico,
Sterman
Bonciani et al.

- Soft gluons transfer colour to the final state
 - anomalous dimension matrix
- dependence on scattering angle / relative rapidity of the 2 final particles

↪ Heavy quark production
($b\bar{b}$, $t\bar{t}$)

Bonciani, Cacciari, Mangano,
Nason;
Kidonakis, Vogt

↪ Prompt photons

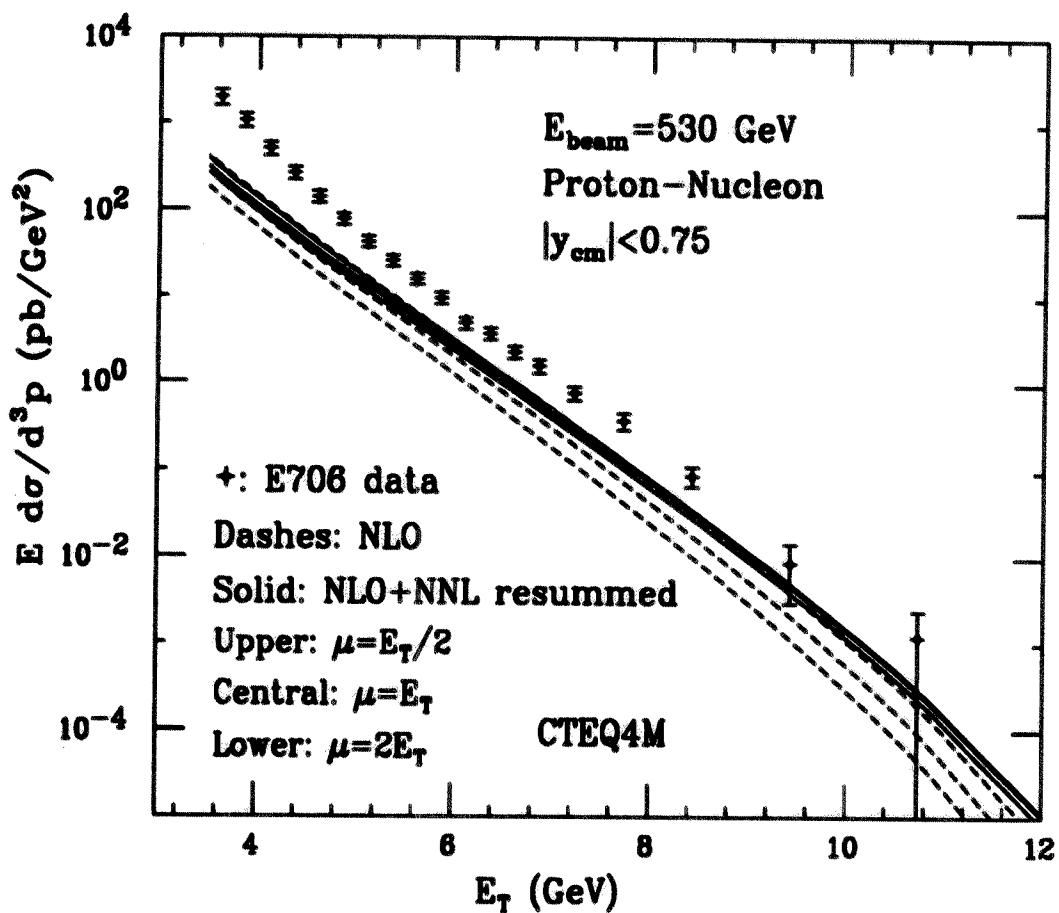
Cacciari, Mangano, Nason,
Oleari, Vogelsang
(Kidonakis)

↪ Di-jets
(formalism complete; not yet
implemented)

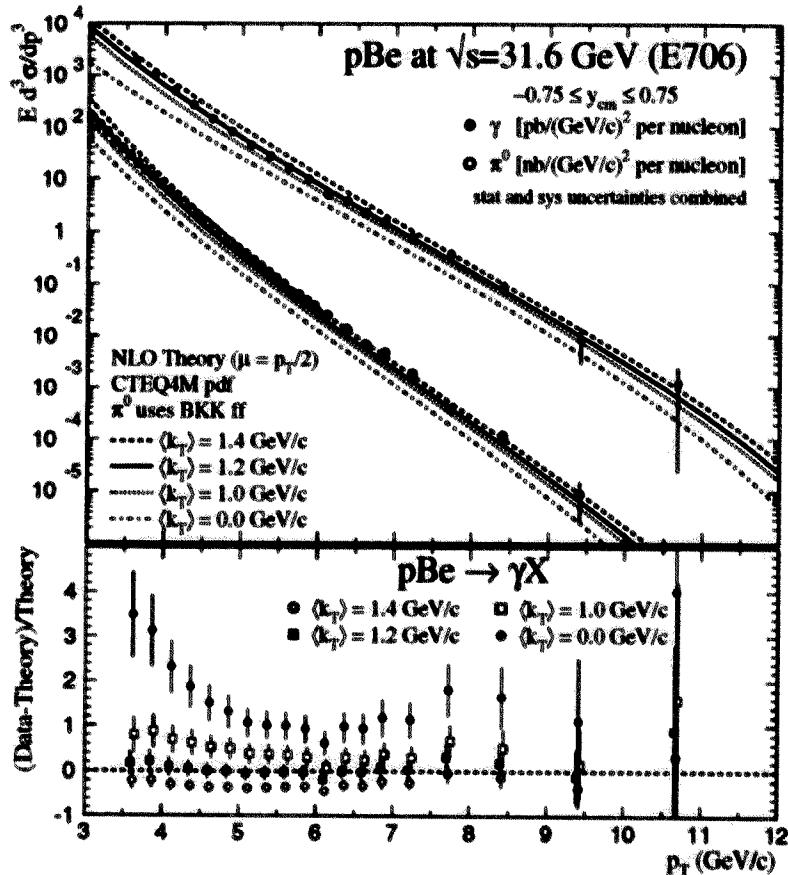
Partonic thresholds

Summary of results

- Resummation effect typically small
[total σ for $b\bar{b}, t\bar{t}$; $d\sigma/dE_T$ for γX]
i.e. within NLO QCD scale variation
- Significant reduction in scale dependence
($\stackrel{?}{=}$ theoretical uncertainty)
- Cannot explain E_T spectrum of prompt photons at low E_T
 - ~ intrinsic k_T ?
 - ~ power corrections?
 - ~ E706 & UA6 consistent with each other?

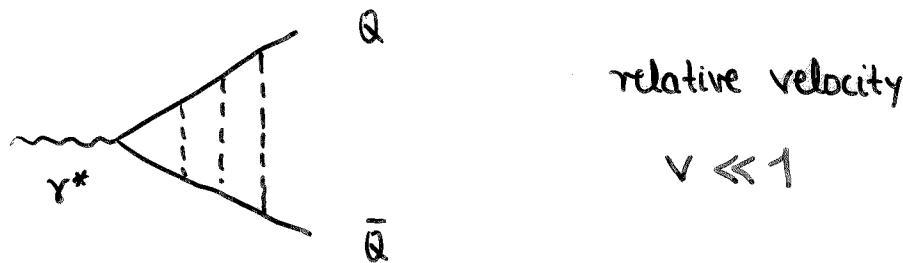


Catani, Mangano, Nason, Oleari, Vogelsang



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Non-relativistic heavy quarks



Strong Coulomb force!

Probes of heavy quark potential, quark masses

Quantum-mechanics concepts - Schrödinger equation,
potential - from field theory ?!

Perturbatively: $[ds \sim v \ll 1]$

$$R_{e^+e^- \rightarrow Q\bar{Q}X} \sim v \cdot \sum_{n=1}^{\infty} \left(\frac{ds}{v} \right)^n \cdot \underbrace{\{ 1; ds, v; ds^2, ds v, v^2; \dots \}}_{\text{NNLO resummation}}$$

[mod. logs of v]

NNLO is sensitive to short-distance physics
($x \sim 1/m_Q$).

Need new, systematic method: non-relativistic effective field theory

Non-relativistic heavy quarks

Classify quarks, gluons as

hard (h), soft (s), potential (p), ultrasoft (us)

$$\mathcal{L}_{\text{QCD}} [Q(h,s,p); g(h,s,p,us)]$$

$$\downarrow \quad m_V < \mu < m$$

Caswell, Lepage
Lepage et al.
Brodsky, Braun, Lepage

$$\mathcal{L}_{\text{NRQCD}} [Q(s,p); g(s,p,us)]$$

$$\downarrow \quad \mu < m_V$$

(Grinstein, Rothstein)
Pineda, Soto
Smirnov, B
Brambilla et al.

$$\mathcal{L}_{\text{PNRQCD}} [Q(p); g(us)]$$

Results on

- $e^+e^- \rightarrow t\bar{t}X$ at NNLO
 - large corrections
 - but $\delta m_t^{\overline{\text{MS}}} \approx 200 \text{ MeV}$ probably ok

Huang, Teubner;
Melnikov, Yelkonin;
Yelkonin;
Signer, Smirnov, B;
Nagano, Oba, Suzuki;
Penin, Pineda/

- m_b from $e^+e^- \rightarrow b\bar{b}X + \gamma(1s)$

$$\bar{m}_b(\bar{m}_b) = 4.23 \pm 0.08 \text{ GeV}$$

(my average)

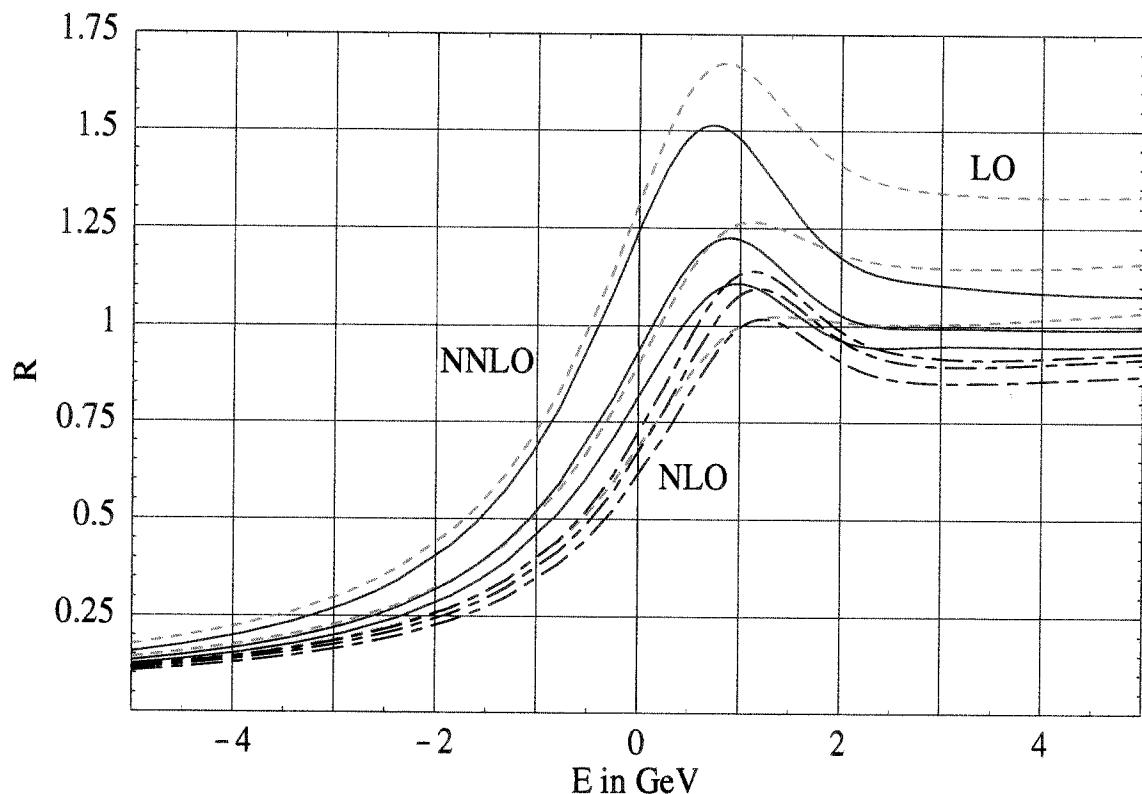
Penin, Pineda;
Huang;
Melnikov, Yelkonin;
Signer, B

Top quark production near threshold

$$E = \sqrt{s} - 2m_{t,\text{PS}}(20 \text{ GeV})$$

$$R = \sigma(e^+e^- \rightarrow \gamma^* \rightarrow t\bar{t}X)/\sigma_{\text{pt}}$$

bands: scale variation, $\mu = 15, 30, 60 \text{ GeV}$



Signer, Smirnov, B

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Small- x

High Energy Limit $s \gg Q^2$

$$\ln \frac{s}{Q^2} = \ln \frac{1}{x_1} + \ln \frac{\hat{s}}{Q^2} + \ln \frac{1}{x_2} \quad [\hat{s} = x_1 x_2 s]$$

Small- x behaviour of DIS
structure functions

High-energy limit of hard
partonic reactions
 $\gamma^* \gamma^*$, forward jets

Summation of $[\alpha_s \ln \frac{s}{Q^2}]^k \Rightarrow$ BFKL equation

■ HERA puzzle : DGLAP works too well

$$P_{gg}(x) \sim \frac{C_F \alpha_s}{\pi} \cdot \frac{1}{x} \quad (+\text{NLO})$$

$$\rightarrow x g(x, \alpha) \sim x g(x, \alpha_0) e^{\text{const.} \sqrt{\alpha_s \ln \frac{1}{x} \ln \frac{Q^2}{\alpha}}} \quad \text{works !}$$

$$P_{gg}(x) \sim \frac{C_F \alpha_s}{\pi} \frac{1}{x} \sum c_n \alpha_s^n \ln^n x$$

$$\rightarrow \frac{d x g(x, \alpha)}{d \ln Q^2} \sim x^{-2} \quad \text{not needed at } x \sim 10^{-6} !$$

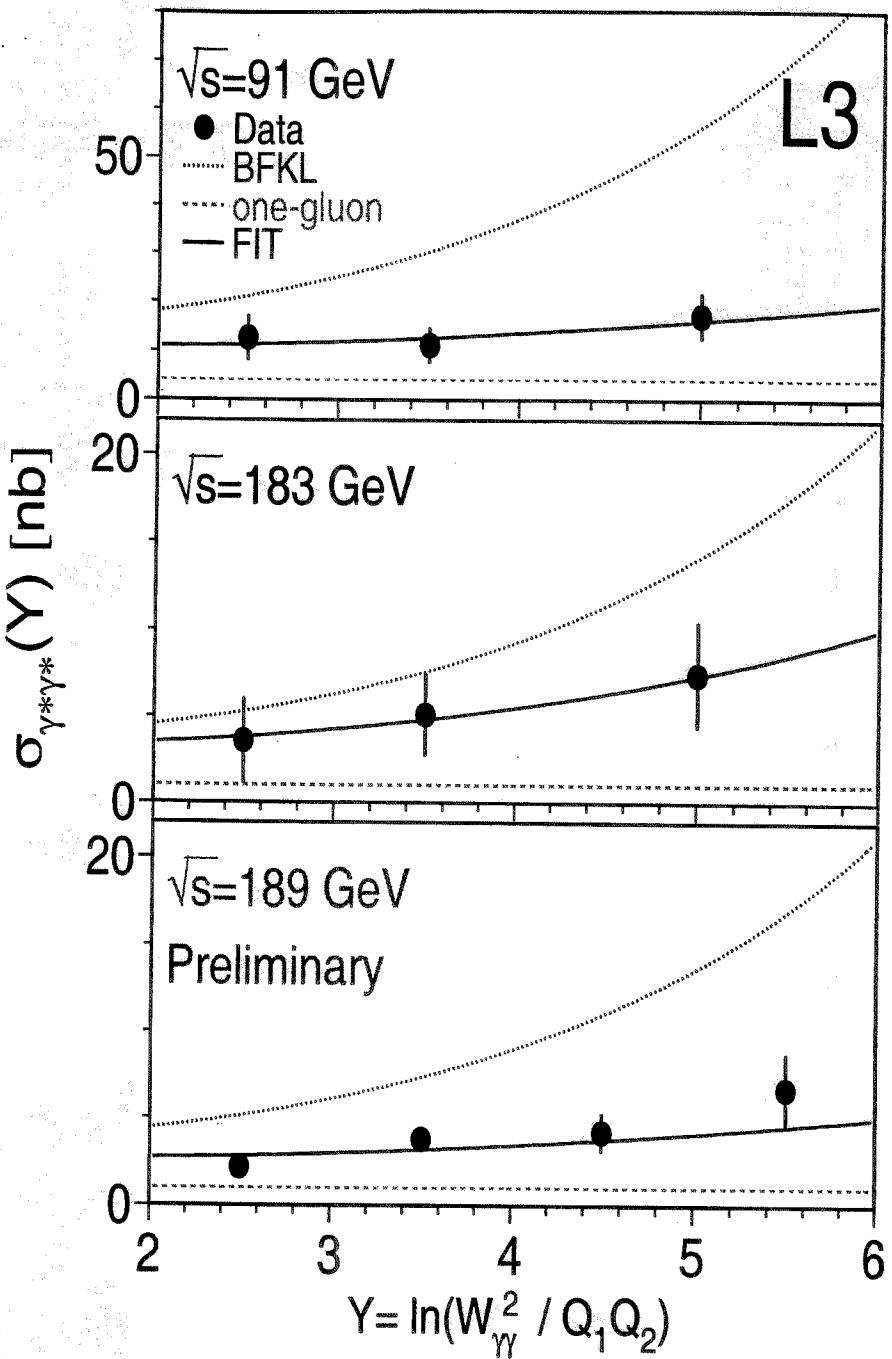
Flexibility in input gluon distribution ?

■ LO BFKL for $\gamma^* \gamma^*$

$$\sigma \sim s^{\frac{1}{2} \alpha_s 4 \ln 2}$$

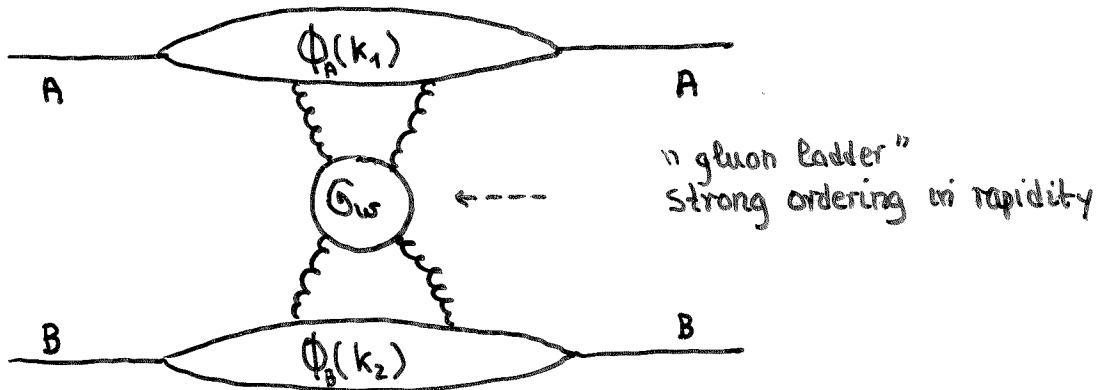
too rapid rise with energy [LS, OPAL]

(perhaps ok for forward $\pi - H$)



See also OPAL.

Small- x



$$\sigma \sim \int \frac{d^2 k_1}{k_1^2} \phi_A(k_1) \frac{d^2 k_2}{k_2^2} \phi_B(k_2) \int \frac{dw}{2\pi i} \left(\frac{s}{k_1 k_2} \right)^w G_W(k_1, k_2)$$

↑
conventional symmetric energy scale

BFKL:

$$w G_W(k_1, k_2) = \delta^{(2)}(k_1 - k_2) + \int \frac{d^2 k}{\pi} K_{W}(k, k') G_W(k, k')$$

new: NLO Kernel

Fadin, Lipatov
(checked by Gafalov, Gorishni;
del Duca, Schmidt)

$$\int d^2 k' K_W(k, k') \left(\frac{k'^2}{k^2} \right)^{\gamma-1} = \bar{\alpha}_s \chi_0(\gamma) \left[1 - \bar{\alpha}_s \beta_0 \ln \frac{k^2}{\mu^2} \right] + \bar{\alpha}_s^2 \chi_1(\gamma)$$

running coupling term

no running coupling

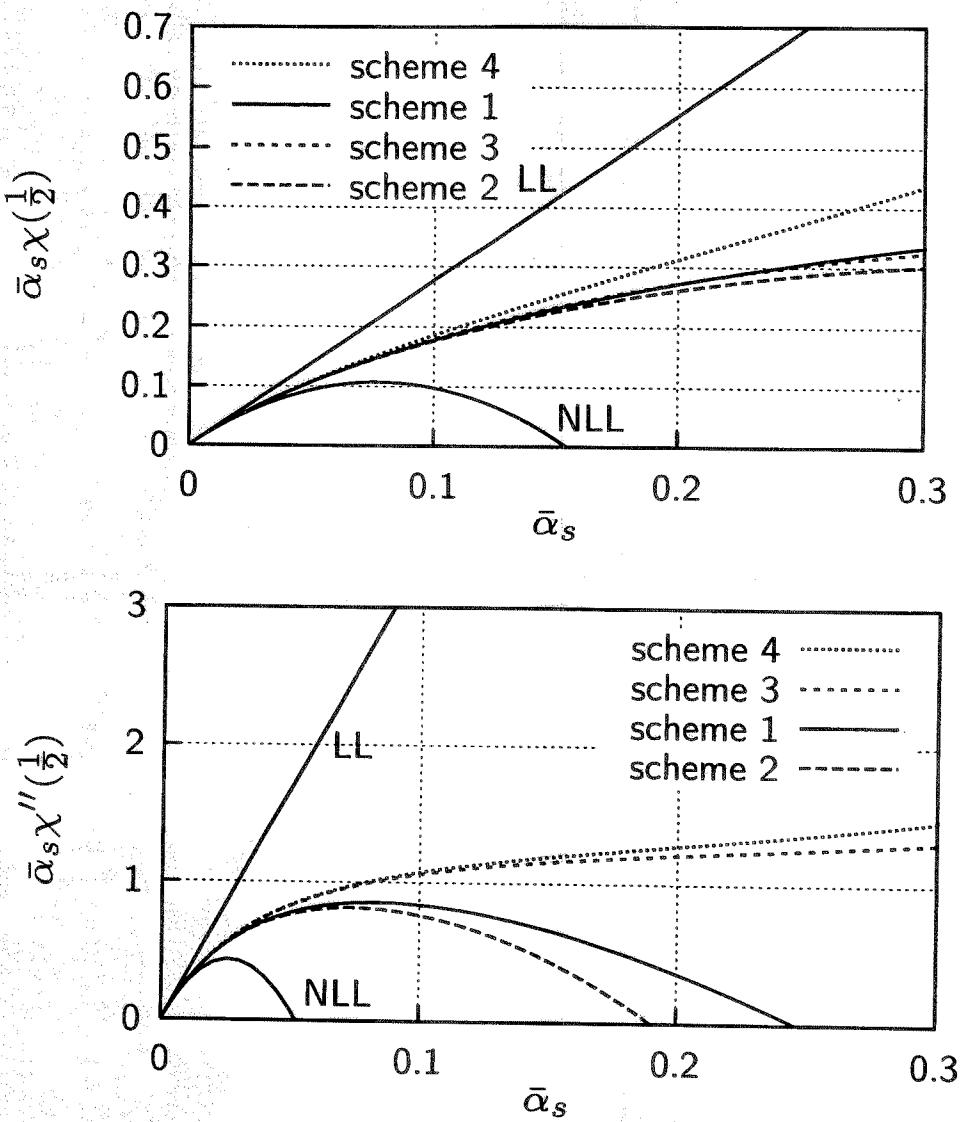
scale-invariant term

$$\xrightarrow[\text{BFKL}]{\Delta p - 1} \bar{\alpha}_s \chi_0(\tfrac{1}{2}) + \bar{\alpha}_s^2 \chi_1(\tfrac{1}{2}) = \bar{\alpha}_s \cdot 4 \ln 2 \left[1 - 6.5 \bar{\alpha}_s \right]$$

Fadin, Lipatov
Gafalov, Gorishni

Huge NLO correction to intercept

[→ oscillatory cross section for $\bar{\alpha}_s \gtrsim 0.05$ θ_{cut}]



Salam

M.Beneke

Small- x

Can well-motivated partial resummations beyond NLO help?

Scale-invariant kernel:

Take $k_1 \gg k_2$ (DIS)

$$ds \ln \frac{k_1^2}{k_2^2} \ln \frac{s}{k_1^2} = ds \ln \frac{k_1^2}{k_2^2} \ln \frac{s}{k_1 k_2} - \frac{1}{2} ds \ln^2 \frac{k_1^2}{k_2^2}$$

LO LO NLO

\leftarrow large even for $\gamma \sim \frac{1}{2}$!

$$\frac{1}{\gamma^3}, \frac{1}{(1-\gamma)^3}$$

\leftarrow double collinear logarithm
singularities in $X_1(\gamma)$

Collinear
Configurations

Sum collinear double logs to all orders

$$X_0(\gamma) \rightarrow X_0^\omega(\gamma) = 2\Psi(1) - \Psi(\gamma + \frac{\omega}{2}) - \Psi(1 - \gamma + \frac{\omega}{2})$$

Satom
(cf. "Lund Kernel")

■ Convergence improves

■ $\alpha_{\text{LP}} - 1$ reduced compared to LO

■ Further support for resummation

can introduce "rapidity veto" $\gamma_{im} - \gamma_i > \Delta$

like a hard cut-off

Schmidt

Δ - dependence small after resummation
NLO moderate for all Δ

Fordham, Ross,
Sobie-Vera

Small- x

running coupling effects

↪ IR and UV diffusion

Even for $k_1 \sim k_2 \sim Q \gg \Lambda_{\text{QCD}}$, this limits the applicability of BFKL:

$$\ln \frac{s}{Q^2} \leq \frac{\pi}{42 g(3) b_0^2} \cdot \frac{1}{d_s(Q)}^3$$

Mueller,
Korchemsky

DIS : $Q \sim k_1 \gg k_2 \sim \Lambda_{\text{QCD}}$

$$G_{15}(k_1, k_2) = F_w(k_1) \cdot F_w(k_2) + O(\frac{k_2^2}{k_1^2})$$

evolution

factorise in input distribution
may contain the "true" pomeron
singularity α_P

$$\frac{\partial F_i}{\partial \ln Q^2} = P_{ij} F_j$$

"physical anomalous dimension"

effective scale of d_s :

$$d_s^{\text{eff}} \sim \frac{1}{\beta_0 \left[\ln \frac{Q^2}{\mu^2} + 3.6 (\bar{\epsilon}(Q) \ln \frac{1}{x})^{1/2} \right]}$$

Thorne
(using BLM)

decreases as $x \rightarrow 0$ because P_{ij} is sensitive only to
UV diffusion

improved fit (compared to MRST) of structure fn data is
claimed!

Small- x

Combine running coupling + energy scale + collinear behaviour

⇒ "Renormalization group improved small- x equation"

Ciafaloni,
Cirigliano,
Salam

Resum terms beyond NLO such that

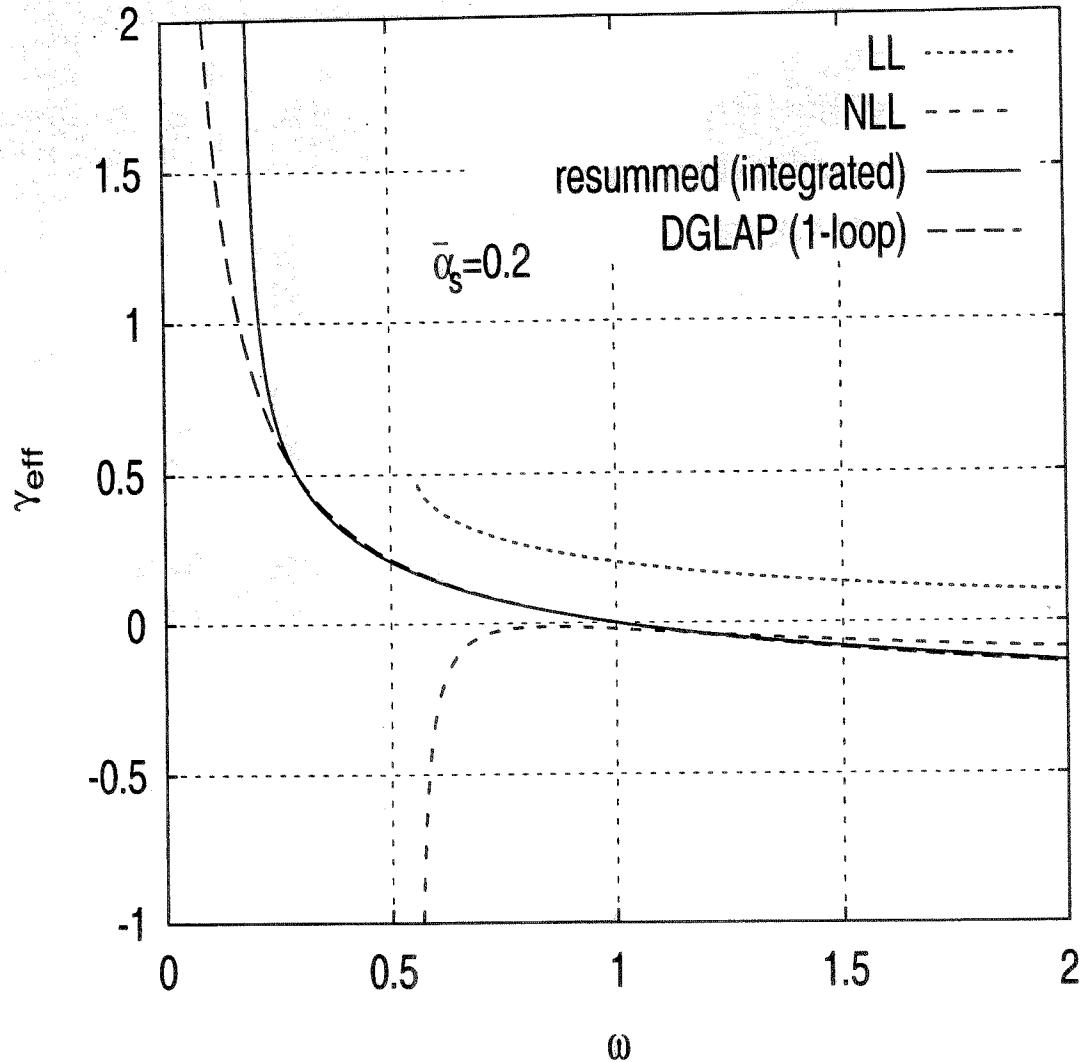
- (1-loop) running coupling treated exactly
- all collinear logs treated correctly, i.e.
LO DGLAP incorporated

$$\gamma_{gg}(w) = \alpha_s A_1(w) + \dots$$

→ energy conservation maintained : $\gamma_{gg}(1) = 0$

↪ resummed anomalous dimension close to
DGLAP down to rather small w (\sim moments)

↪ claims reasonable bounds on (hard) pomeron
intercept $\alpha_{1P} - 1 \sim 0.17 - 0.27$ for $\alpha_s = 0.2$



Ciafaloni, Colferai, Salam

Effective gluon anomalous dimension in moment space.

Momentum conservation: $\gamma_{\text{eff}}(1) = 0$

Small -x

Summary

- NLO correction known
- Consistency with collinear behaviour is crucial (\rightarrow beyond NLO)
- Results point into the right direction

Warning:

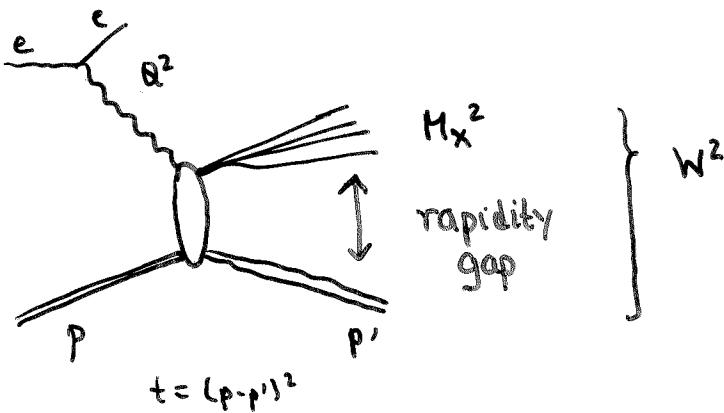
Theorists need some time to work this out right!

- ↪ no NLO impact factors $\Phi_R(k)$ known so far (need γ^*)
- ↪ more work on symmetric processes ($\gamma^*\gamma^*$) needed

Novel Factorization Theorems

- Hard Diffractive Scattering
- Diffractive Vector Meson Production
and Virtual Compton Scattering
- Exclusive B Decays

Hard diffractive scattering



$$x = \frac{Q^2}{Q^2 + W^2}$$

$$\xi = \frac{Q^2 + M_X^2}{Q^2 + W^2} = x_{IP}$$

= momentum fraction p
loses to the colour
neutral system X

$$\beta = x/\xi$$

Hard diffraction is a leading twist phenomenon

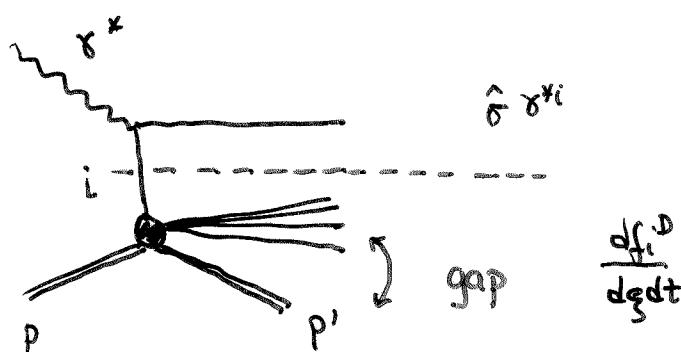
Factorization theorem

$$\frac{d\sigma^{Y^* p \rightarrow p' X}}{d\xi dt} = \sum_{i=q,g} \int_x^y dy \hat{\sigma}^{Y^* i}_{(x, Q^2, \xi, t)} \frac{df_i(y, \xi, t; \mu)}{d\xi dt}$$

Trentadue, Venugopalan,
Berenru, Soper;
Graudini, Trentadue,
Venugopalan;
Collins

"diffractive parton distribution"

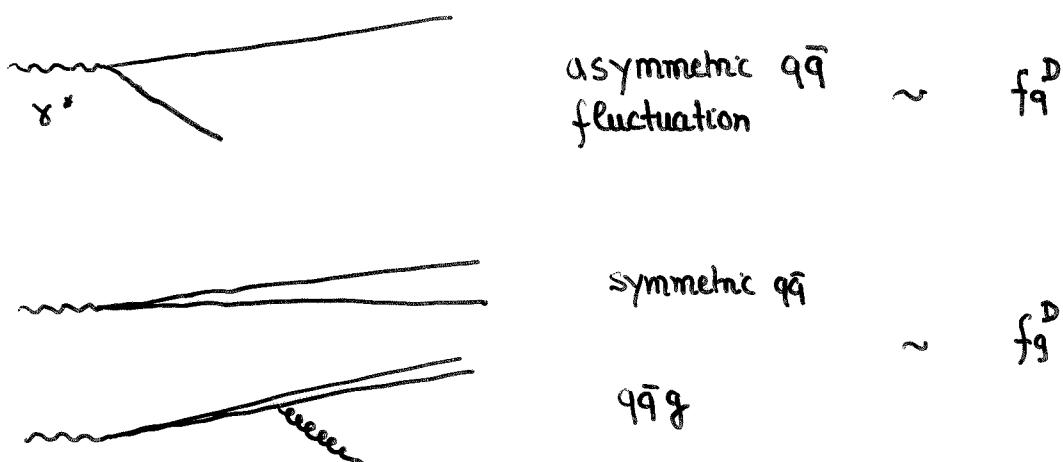
= prob. to find parton i in proton
with momentum fraction x under the
condition that the proton stays intact
and loses momentum fraction ξ



close analogy
with inclusive
DIS !

Hard diffractive scattering

- no reference to rapidity gap;
 $\Delta y \sim \ln \frac{1}{x}$ kinematically for small ξ
- no reference to Regge factorization & the pomeron
- Physical picture in the proton rest frame:
"aligned jet model" Bjorken;
Bjorken, Kogut



fluctuation with large transverse size scatters on the proton

- short-distance cross section $\hat{\sigma}^{x_i} +$ evolution of f_i^D
identical to inclusive DIS
 \Rightarrow characteristics of hard diffraction is in the input distribution

Hard diffractive scattering

Models for diffractive parton distributions

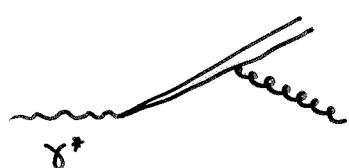
Regge factorization

$$x \frac{df_i(x, \xi, t)}{dt} = f_{ip}(\xi, t) \cdot \beta^{\frac{1}{1-2\alpha_{ip}(t)}} \cdot \beta(\xi)$$

pomeron flux

partonic content of the pomeron

popular, but not QCD!



models for the proton field

large proton
(semiclassical treatment)

Buchmüller, Hebecker :

+ Mc Dermott ;

Hebecker ;

Buchmüller, Gersmann,
Hebecker

small-size proton
modelled by



Hautmann, Kanzel, Saylor

[similar to 2-gluon exchange
treatment

Nikolaev, Burkhardt : Abramowicz
et al. ; Bartels, Wüsthoff]

Hard diffraction

Both models give similar results !

$$\xi \frac{df_i}{d\xi dt} \sim 1/\xi \quad \text{for } \xi \rightarrow 0$$

$$\begin{array}{lll} \beta \rightarrow 0 & \beta \rightarrow 1 \quad (t=0) \\ q & 1 & 1-\beta \\ g & 1/\beta & (1-\beta)^2 \end{array}$$

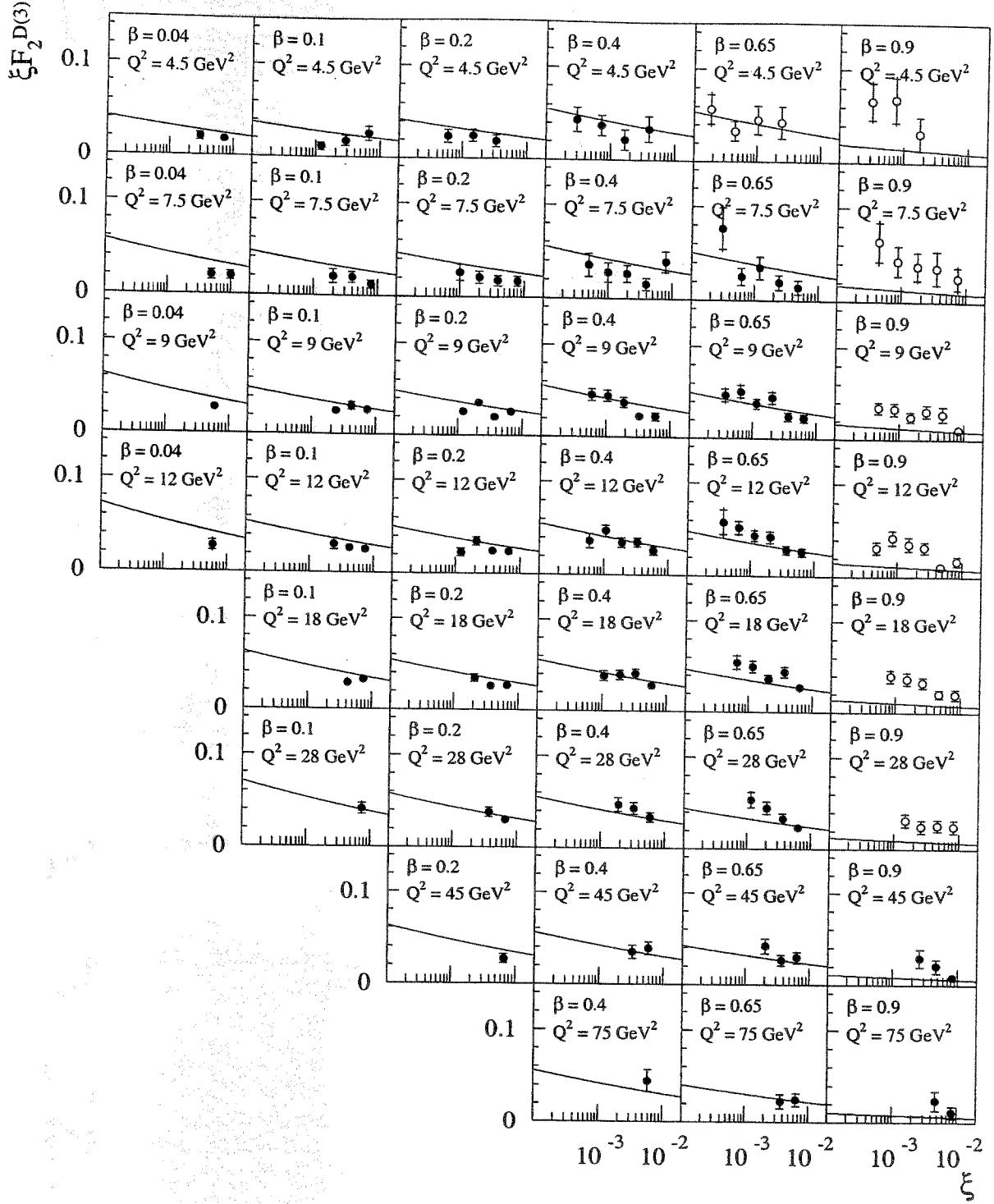
f_8/f_q enhanced by a large colour factor

↪ Q^2 -behaviour of $F_2^{\text{inel.}}$ and F_2^{D3} is different

F_2^{D3} grows with Q^2 for $\beta \leq 0.5$

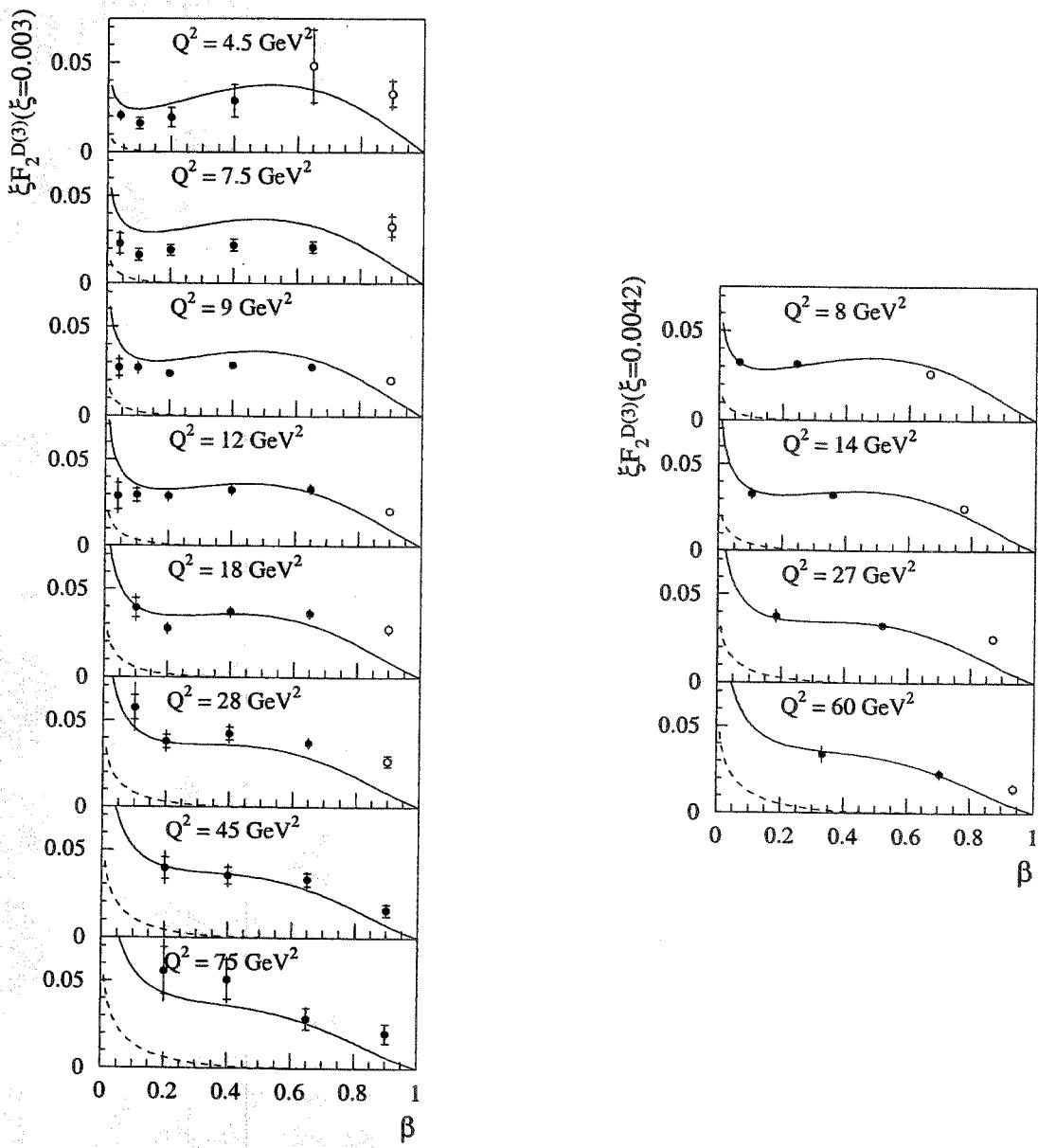
↪ simple models reproduce gross features of the data

test of γ^* wave function + aligned jet picture rather than probe of proton structure !



Buchmüller, Gehrman, Hebecker

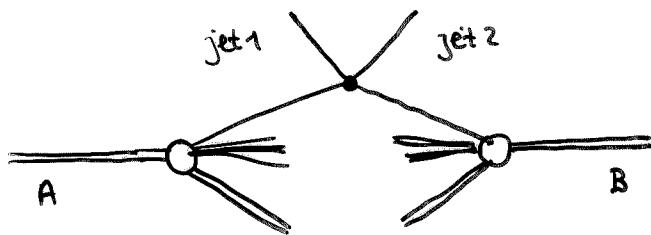
The diffractive structure function $F_2^{D(3)}(\xi, \beta, Q^2)$ at small ξ in the semiclassical approach. H1 data.



Buchmüller, Gehrmann, Hebecker

Scaling violation of $F_2^{D(3)}$ in the semiclassical approach vs H1 data (left) and ZEUS data (right)

Hard diffractive scattering



Hadron-hadron collisions:

- gap between jets
- gap between jet and elastically scattered hadron

No factorisation

Beren, Sept

Diffractive systems cross each other

→ gap survival probability P_S

Bjorken

Suggestion: Define gap by energy flow rather than particle multiplicity

Stirling,
Sudakov

$$\Rightarrow P_S = 1 + O\left(\frac{\Lambda_{\text{QCD}}}{Q_c}\right)$$

energy flow in
gap

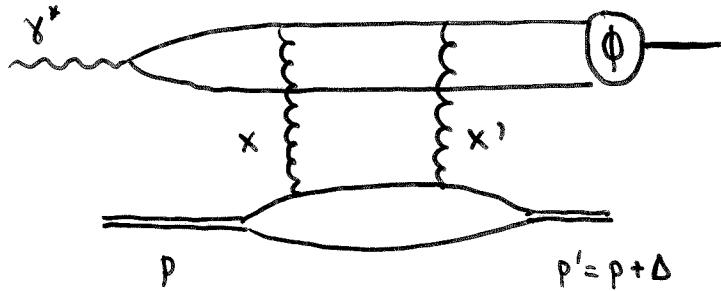
for gaps between jets

$$\Lambda_{\text{QCD}} \ll Q_c \ll |\vec{t}|$$

Skewed processes

Factorisation theorems for

(a) Diffractive vector meson production



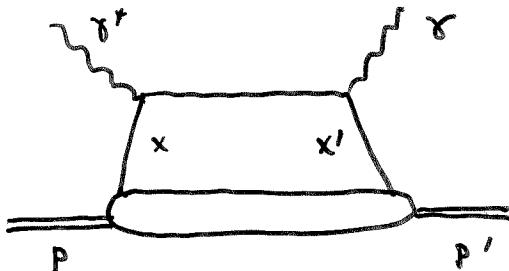
Collins, Frankfurt,
Strikman

(Ryskov;
Brodsky et al.)

$$\gamma_L (\Omega^2 \gg \Lambda_{\text{QCD}}^2) \rightarrow \rho_L$$

$$\gamma (\text{any } \Omega^2) \rightarrow J/\psi, \gamma, \dots$$

(b) Deeply virtual Compton scattering



Ji
fixed problem
(Collins, Frankfurt)

Requires a new type of parton distribution

"skewed (off-diagonal, non-forward)
parton distribution"

Skewed processes

$$p^+ \int \frac{dz^-}{2\pi} e^{ix p^+ z^-} \langle p' | \bar{\Psi}(0) \gamma^+ \Psi(z^-) | p \rangle$$



$p' \rightarrow p$: inclusive pdf

$z^- \rightarrow 0$ (1st moment) : form factor

Hybrid properties in evolution:

$x' > 0$: DGLAP type

$x' < 0$: ERBL type



emission of
q-q̄ pair
for $x' < 0$

Effects of skewedness:

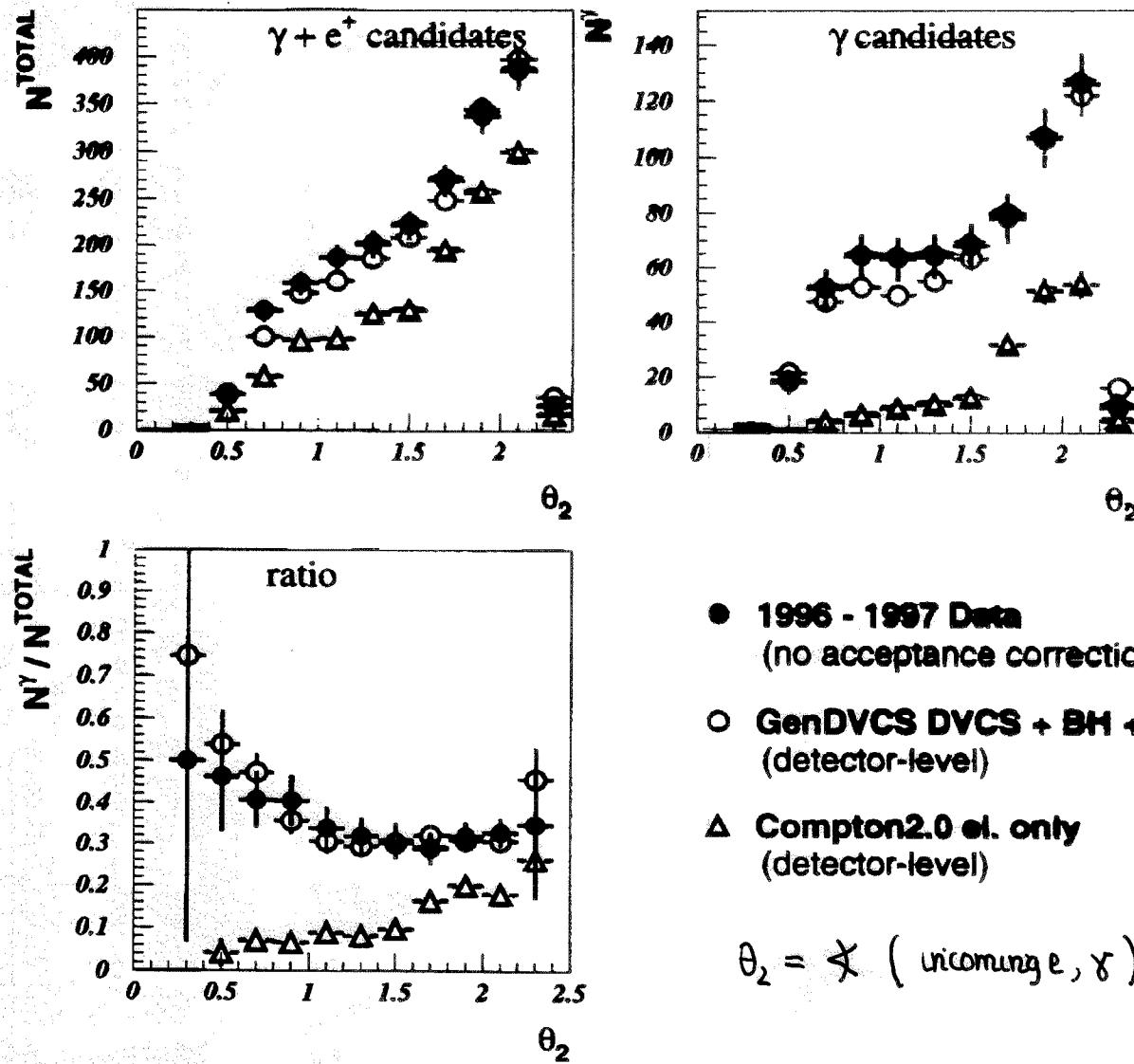
- γ photoproduction at HERA

Frankfurt, Klempt, Strikman

- Evidence for deeply virtual Compton scattering [Bauer]

DVCS - θ_2 DISTRIBUTIONS

ZEUS 1996/97 Preliminary



→ Appears to be clear signal for DVCS!

But, processes like $ep \rightarrow ep\pi^0$, $ep \rightarrow ep\pi^0\pi^0$,
 $ep \rightarrow ep\pi^0\eta$, ... potentially fake this signal.

Exclusive B decays

$$A(B \rightarrow M_1 M_2) = A_1 e^{iS_1} e^{i\delta_{W1}} + A_2 e^{iS_2} e^{i\delta_{W2}} + \dots$$

$\underbrace{\phantom{A_1 e^{iS_1} e^{i\delta_{W1}}}}_{\text{QCD}}$ $\underbrace{\phantom{A_2 e^{iS_2} e^{i\delta_{W2}}}}_{\text{NP}}$

Exclusive process with large momentum transfer $m_b^2 \sim 25 \text{ GeV}^2$!

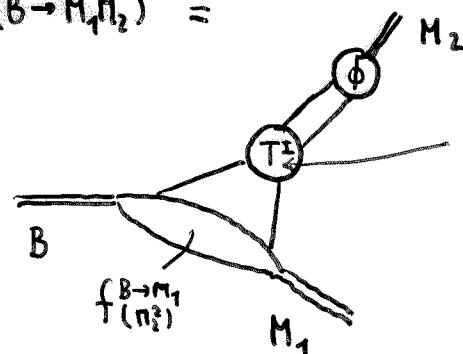
But: Standard methods (Brodsky-Lepage) don't work,
because B meson contains a soft spectator

Factorization theorem

Brodsky, Neustroev,
Sudakov, B

(Politzer, Wilce
for D π)

$$A(B \rightarrow M_1 M_2) =$$



$$\cancel{V} + \cancel{A} + \dots$$

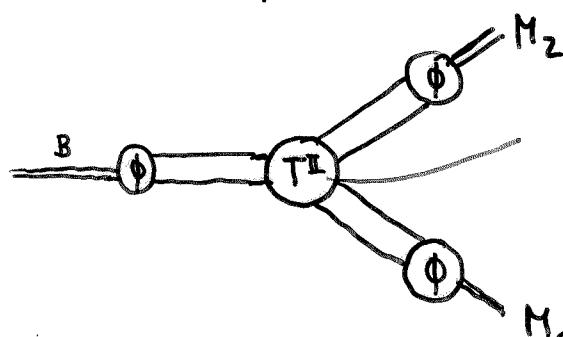
$$+ \cancel{b} u \bar{u} + O(\epsilon_s^2)$$

$$f_{(M_1^2)}^{B \rightarrow M_1} \cdot f_{M_2} \cdot \int du T^I(u) \Phi_{M_2}(u)$$

light-cone distribution amplitude

+

perturbative kernel



$$\cancel{V} + \cancel{A} + O(\epsilon_s^2)$$

Brodsky-Lepage type term

$$\int d\zeta du dv T^I(s, u, v) \Phi_B(s) \Phi_{M_1}(u) \Phi_{M_2}(v)$$

$$+ O\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)$$

Exclusive B decays

→ in the heavy quark limit non-factorizable corrections are calculable

need: form factors

light-cone distribution amplitudes

(use pion form factor, $\gamma\gamma^*\pi$ etc.)

■ Factorization proof to all orders has still to be given

■ Main limitation: Λ_{QCD}/m_b corrections

↳ great potential for B factories; but applications need to be examined carefully

↳ take B decays seriously as a hard exclusive process

Light-cone properties of B meson are basically unexplored!

Summary

An incredible variety of phenomena is described quantitatively by QCD

- Progress in precision

- higher loops, resummation, power corrections

-

- Progress in scope

- new factorization theorems allow us to

- study new processes quantitatively