Probing Higher Curvature Gravity in Extra Dimensions at Colliders

Higher Curvature Terms:

⇒ What are they and when are they important?

Where do we see them?

⇒ Modifications in 'traditional' extra dimensional model signatures...

• 'Large' Extra Dimensions (ADD) ⇒ BH
  • 'Warped' Randall-Sundrum model (RS)

Summary + Conclusions

JHEP1(2005)028
hep-ph/0503163

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5/05
\[ S = \int d^{4+n}x \sqrt{g} \left\{ \frac{M_x^{n+2}}{2} R - \Lambda + ??? \right\} \leq \text{GRAVITY} \]

*Einstein-Hilbert (EH) action*

- \( M_x = \text{fundamental scale} \)
- \( R = \text{Ricci curvature scalar} \)
- \( \Lambda = \text{Cosmological constant} \)

**EH is**

- The basis of GR in 4d.....
- The basis for ADD/RS models in extra dimensions
- At best, an effective theory below \( M_x \) (~few TeV ??, \( M_{pl} \) ?)

"??" terms from "UV-completion" (strings?) may be important as Energies approach \( M_x \)...

\[ \Rightarrow \text{What can they be ??} \]

- \( R^2, (\partial R)^2, R^{-3}, R_{AB} R^{BC} R_{CA}, \ldots \) ??

... many, many possibilities

- We need guidance !!!
Unitary / ghost-free theory \[\Rightarrow\] no derivatives of the metric > 1st (2nd) in the action (e.g., of motion) \[\Rightarrow\] Lagrange's Eq's.

- Produces 'benign' modifications to Einstein Eq's:

\[ R_{AB} - \frac{1}{2} g_{AB} R + L_{AB} = \frac{1}{M_{n+2}} T_{AB}. \]

Extra terms are symmetric + divergence free...

- String Motivation \[\Rightarrow\] lowest order terms arise from string expansion... [Zwiebach '84], [Zumino '86]

\[ \Rightarrow \text{Unique Solution! The action is a sum of} \]

Lovelock invariants at low energies...

\[ L_m \sim S_{C_1 D_1 \ldots C_m D_m} R_{A_1 B_1} \ldots R_{A_m B_m} \]

totally antisymmetric

Kronecker

Riemann

Curvature

Tensor

Too many terms? Not really...
In $D = 4+n$ MOST $\mathcal{L}_m$ are zero! ... only $\mathcal{L}_m$ with $D \geq 2m+1$ are 'dynamical' = contribute to Einstein's e.g.s., i.e.,

\[
\begin{align*}
D = 4 & \quad \{ \begin{array}{l}
\mathcal{L}_0 = a \text{ constant (W)} \\
\mathcal{L}_1 = R
\end{array} \} \text{ EH!} \\
D = 5, 6 & \quad \{ \begin{array}{l}
\mathcal{L}_2 = R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD} \\
\text{(Gauss-Bonnet)}
\end{array} \} \\
D = 9, 10 & \quad \{ \begin{array}{l}
\mathcal{L}_3 = 8 \text{ terms}
\end{array} \}
\end{align*}
\]

\ldots etc, etc...

\text{EH is just the first two terms of a general expansion...}

\ldots it is the UNIQUE Lovelock action in $D=4$!

\ldots in $D=5$, $R + \frac{\alpha}{M_x^2} \mathcal{L}_2$ is unique (RS)

\[\sum_{m} \alpha_m \mathcal{L}_m: \text{ often used in literature as toy models to}
\text{ probe quantum/stringy corrections to EH...}
\text{in Extra Dimensions...} \]
The Love-Lox invariant grow rapidly in complexity. . .
Are the $L_m$ important in ADD/RS models?

\[ S = \int d^{n+2}x \sqrt{g} \left\{ \frac{M_x^{n+2}}{2} \left[ R \frac{\alpha}{M_x^2} L_2 + \frac{B}{M_x^4} L_3 + \frac{x}{M_x^6} L_4 \right] - A \right\} \]

- $\alpha, \beta, \gamma$ = dimensionless constants

- Since $L_m \sim R^n$, Lovelock 'corrections' are potentially large when $R/M_x^2$ is big...

⇒ When does this happen??

Look at models... in $\{\text{ADD} \}$, but first...

How large are $\alpha, \beta, \gamma$? If PT holds, then:

\[
\begin{align*}
\alpha &\sim 1/D^2 \sim \text{few} \cdot 10^{-2} \\
\beta &\sim 1/D^4 \sim \text{few} \cdot 10^{-4} \\
\gamma &\sim 1/D^6 \sim 10^{-5}
\end{align*}
\]

... keep these values in mind for the $\text{ADD}$ case below...
ADD Basics \{ Arkani-Hamed, Dimopoulos, Dvali \} ('98)

- Bulk gravity in \( D = 4 + n \) dimensions
- \( n \) extra dims usually compactified on a Torus, i.e., "flat" (conformal):

\[ M_{\text{pl}} = V_n M_{x}^{n+2} \]

true fundamental scale \( \sim \text{TeV} \)

\[ V_n = \frac{1}{G_{\text{Newton}}} \]

Vol. of compact dims \( \sim (2\pi R)^n \)

- \( n = 1 \) \( R \sim 10^{15} \text{ cm} \) (too big)
- \( n = 2 \) \( R \sim 100 \mu \text{m} \) (Table top)
- \( n = 3 \) \( \sim 10^{-9} \text{ m} \)

\[ L = -\frac{1}{M_{\text{pl}}} \sum \frac{h_{\mu\nu}^{(n)}}{n} T_{\mu\nu} \]

stress tensor for SM matter

\[ M_{\text{KK}} = \frac{n^2}{R^2} \]

- Tiny masses
- Tiny spacing

\[ \rightarrow \text{ emission signature} \]

- \( h_{\mu\nu} \) - many states appears as \( \tau_{+} \)
- \( \bar{h}_{\mu\nu} \) m
- \( e_{\mu\nu} \)
- \( e_{\mu\nu} \)
- \( \tau_{+} \) monojet + \( \tau_{+} \) at LHC
- \( \bar{h}_{\mu\nu} \) m
- \( \tau_{+} \) m + 'nothing' at ILC

you are here

3-brane
The signal is the little guy on top...

\[ \delta = 4 \quad M_D = 5 \text{ TeV} \]

**Excess Missing Energy at LHC**

\[ \sqrt{s} = 14 \text{ TeV} \]

**LHC:**
- \( jW(e\nu), jW(\mu\nu) \)
- \( jW(\tau\nu) \)
- \( jZ(\nu\nu) \)
- **Signal**

**Figure 1:** Missing energy spectrum at the LHC.

Understand your backgrounds!!
- exchange
  Signature:
  \[ \bar{q} \rightarrow h^{(n)} \rightarrow l^+ , \quad e^+ \rightarrow h^{(n)} \rightarrow \bar{b} \]
  \[ \theta = \frac{4 \lambda}{M_H} T^{(1)}_{\nu} T^{(2)} \]

- These are not influenced by higher-curvature terms \textbf{EVEN if not compactified on a} \( T^n \) torus... \([ (R_c M_s)^2 > 71 ] \)

- What's Left? Black Holes!! Why?
  - \( \frac{R}{M_s^2} \) is large near BH's...

- BH's in extra dims were studied long ago...
  - e.g., Schwarzschild-like soln's...
    - Boulware+Deser '85
    - Wheeler '86
    - Whitt '88
    - Wiltshire '86 +...

- TeV-scale BH
  - Banko+Fischler '99
  - Dimopoulos+Landsberg '01
  - Giddings+Thomas '01

Reviews:
  - Ikanti '04
  - Hossenfelder '04
Black Holes on Demand

Scientists are exploring the possibility of producing miniature black holes on demand by smashing particles together. Their plans hinge on the theory that the universe contains more than the three dimensions of everyday life. Here's the idea:

Particles collide in three dimensional space, shown below as a flat plane.

As the particles approach in a particle accelerator, their gravitational attraction increases steadily.

When the particles are extremely close, they may enter space with more dimensions, shown above as a cube.

The extra dimensions would allow gravity to increase more rapidly so a black hole can form.

Such a black hole would immediately evaporate, sending out a unique pattern of radiation.
Once produced, the black holes will undergo an evaporation process whose thermal properties carry information about the parameters $M_f$ and $d$. An analysis of the evaporation will therefore offer the possibility to extract knowledge about the topology of our space time and the underlying theory.

The evaporation process can be categorized in three characteristic stages [36], see also the illustration in Figure 8:

1. **Balding Phase**: In this phase the black hole radiates away the multipole moments it has inherited from the initial configuration, and settles down in a hairless state. During this stage, a certain fraction of the initial mass will be lost in gravitational radiation.

2. **Evaporation Phase**: The evaporation phase starts with a spin down phase in which the Hawking radiation carries away the angular momentum, after which it proceeds with emission of thermally distributed quanta until the black hole reaches Planck mass. The radiation spectrum contains all Standard Model particles, which are emitted on our brane, as well as gravitons, which are also emitted into the extra dimensions. It is expected that most of the initial energy is emitted in during this phase in Standard Model particles.

3. **Planck Phase**: Once the black hole has reached a mass close to the Planck mass, it falls into the regime of quantum gravity and predictions become increasingly difficult. It is generally assumed that the black hole will either completely decay in some last few Standard Model particles or a stable remnant will be left, which carries away the remaining energy.

\[ \chi_{BH} \leq 10^{-20} \text{ see here} \]
Black Hole Forms

\[ \sqrt{s} > M_\chi \]

Collide Beams

\[ \text{gravitational radiation} \]

Step-function Turn-on !!

Issues:

\[ \hat{\sigma} = A_n \pi R_s^2 \sim \frac{1}{M_k} \left( \frac{M_{BH}}{M_\chi} \right)^{\frac{2}{n+1}} \]

\{ This is huge if \( M_\chi \sim 1 \text{TeV} \) or so \}

- What is \( A_n \)? (suppression?)
- What is \( M_{BH}/\sqrt{s} \)? ("efficiency"?)

Yoshino + Rychov (hep-th/0503171):

\[ \Rightarrow A_n \text{ is 1.5 (D=5), 3.2 (D=11) from detailed sim.} \]

\[ \Rightarrow M_{BH}/\sqrt{s} = 0.60-0.75 \]

Cardoso, Berti + Cavaglia (hep-ph/0505125):

- multiple techniques to obtain \( M_{BH}/\sqrt{s} \) ... for \( D=5-10 \)
  1) 0.40 - 0.65
  2) 0.99 - 1.0
  3) 0.90 - 0.92

Controversy remains + lots of work needs doing..

\[ \Rightarrow \text{Here I assume } A_n = 1, \ M_{BH}/\sqrt{s} = 1 \text{ as is usually done in collider analyses...} \]
The unusual nature of BH events should make them relatively easy to spot at the LHC.
It is relatively easy to extract BH properties from kinematic distributions.

\[ \text{LHC} \quad \{ \begin{align*}
M_{BH} &= 8 \text{ TeV} \\
100 \text{ fb}^{-1}
\end{align*} \]

CHARYBDIS
Cambridge Group

**Leading Jet**

- **Event**
  - \( n=2 \)
  - \( n=6 \)

- **\( P_T \) Jet 1**
  - Range: 0 to 4000

**Next to leading jet**

- **Event**
  - \( n=6 \)

- **\( P_T \) Jet 2**
  - Range: 0 to 300

**Event**

- \( n=2 \)
- \( n=6 \)

- **\( P_T \) Lepton**
  - Range: 0 to 4000

- **Missing \( P_T \)**
  - Range: 0 to 15
BH mass reconstruction at LHC from visible decay products... quite reasonable...

5 TeV BH

Event

100000

50000

-2000 -1000 0 1000 2000

M^{\text{rec.}}_{BH} - M^{\text{True}}_{BH} (GeV)

Constant 0.8685E+05
Mean -83.23
Sigma 202.1

CHARYBDIS
HERWIG+
ATLFAST

Cambridge group
hep-ph/0411022
Issues II: Do BH decay more to brane or bulk modes (in ADD)??

\[
\begin{align*}
\frac{dN}{dt} &= N_3 R_s^2 T^4 \cdot n_{\text{brane}} \quad \text{number of brane (60) and bulk (1) modes} \\
\frac{dM}{dt} &= N_3 R_s^{2+n} T^{4+n} \cdot n_{\text{bulk}}
\end{align*}
\]

\[\Rightarrow \quad \frac{n_{\text{brane}}}{n_{\text{bulk}}} \approx 250 - 720 \; \checkmark \quad \text{SM modes dominate!}
\]

Now

\[R_s = R_s(\alpha, \beta, \delta), \quad T_{\text{BH}} = T_{\text{BH}}(\alpha, \beta, \delta) \quad \text{etc}
\]

\[\Rightarrow \quad \text{Quantitative and Qualitative Changes}
\]

E.g.,

- for \( n = 3, \beta \neq 0 \) NO BH can form below a critical minimum mass \( \delta = 5, \delta \neq 0 \)

\[\Rightarrow \text{removes unphysical step-function turn-on...}
\]

Furthermore for \( M_{\text{BH}} = M_{\text{crit}} \), BH are \text{STABLE} in these cases \( \Rightarrow \) “Planck phase” info?
Shift in Schwarzschild radius

$\frac{R}{R_0}$

$\alpha \neq 0$ only
Large $\alpha$ reduces $R$ compared to EH
(hence cross-section)

$\alpha$ vs. $\frac{m_{BH}}{m_*}$
Solid: $\frac{m_{BH}}{m_*} = 2$
Dash: $\frac{m_{BH}}{m_*} = 5$
Dot: $\frac{m_{BH}}{m_*} = 8$
$O(1)$ Temperature changes

$\alpha \neq 0$
only

$n = 2$

$T/T_0$

$\alpha$

$10^{-4}$ $10^{-3}$ $10^{-2}$ $10^{-1}$ $10^0$
Zero Radius

\[
\frac{R}{R_0}
\]

\[
\beta \neq 0 \text{ only}
\]

\[
\beta
\]

\[
h = 20
\]

\[
h = 6
\]

\[
h = 4
\]

\[
h = 3
\]
$\beta \neq 0$ only

$\frac{T}{T_0}$

$n = 3$

$n = 6$

$n = 20$

Temperature

$\beta$
'Simple' Example: \( \beta \neq 0 \) \( \omega \) \( n = 3 \) Then:

\[
R_s M_x = \left\{ \frac{M_{BH}/M_x}{\pi^{3/2}} - 24 \beta \right\}^{1/4} \Rightarrow
\]

Unless \( M_{BH} > M_{\text{crit}} = 60\pi^{3/2} \beta M_x \), no BH will form!

\( \sim O(1) \) \( \Rightarrow \) Threshold!

**Lifetime:** \( \frac{dM}{dt} \sim (\text{Area}) (\text{Temp})^4 \)

\[
\sim \left( \frac{M_{BH} - M_{\text{crit}}}{M_{BH} + 2M_{\text{crit}}} \right)^{7/2} \frac{1}{(M_{BH} + 2M_{\text{crit}})^4} \Rightarrow
\]

- For any \( M_{BH} > M_{\text{crit}} \), this is \( \infty \)! Why?
- Lovelock BH can **cool** as they lose mass unlike EH BH...
- Other scenarios that try to capture some 'quantum' BH aspects also lead to thresholds & long-lived BH...

  i) running \( G_N \): Bonanno + Reuter, hep-th/0002196
  ii) loop QG: Bojowald, Goswami, Maartens + Singh, gr-qc/0503041
  iii) finite length models: Cavaglia, Das + Maartens: hep-ph/0305222
  Hossenfelder: hep-th/0404252
a real threshold

LHC

M (GeV)

 EVENTS/BIN

10^0  10^1  10^2  10^3  10^4  10^5

1000  2000  3000  4000

... need precision to probe parameters

Bh at LHC

100 fb^{-1}
$e^+ e^-$

$\sigma M_*^2$

$\left( \frac{M_{BH}}{M_*} \right)$

$\sqrt{s} / m_*$

**Step-Function**

$n = 3$

$\beta = 0.0005$

**Note**

If $M_x = M_* \
\rightarrow 39 \text{ pb}$
Threshold shapes will tell us $(\alpha, \beta, \gamma)$...

\[
\sqrt{s}/m_w
\]
Crude Estimate of BH lifetime...

BH in a uniform medium ($g = 3\sigma_0/cm^3$)

$n = 3$
$\beta = 0.0005$

$M_{BH} = m_{crit} + 8$

Initially $M_{BH} = 1.5 m_{sun}$
Randall–Sundrum Basics: 1 extra dim.

Two 3-branes
- we live on SM brane
- gravity lives everywhere
- compactified on $S^1/Z_2$

\[ dS^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 \]
- warp factor

\[ \frac{K}{M_{Pl}} \approx 0.01 - 0.1 \]
\( (K \leq M_P) \)

\[ m_{kk,n} = \left( \frac{x}{n} \right)^{k - \pi r c} \]
\[ J_1 (x_n) = 0 \sim \text{few hundred GeV} \]

\[ L_{kk} = -\frac{i}{\Lambda_{kk}} \sum G^{(n)}_{\mu\nu} T_{\mu\nu}^{sm} \]
\[ \Lambda_{kk} = \frac{\Lambda_{Pl}}{e^{-kr c r}} \]

\[ \therefore \] Spin-2 resonances w/ TeV-scale couplings
- couplings identical for all KK levels
- AdS$_5$ bulk $\Rightarrow$ strong curvature $\therefore$ potentially significant higher curvature corrections $!!$, i.e.
graviton resonance production at LHC

\[ \bar{q}q / g g \Rightarrow X \Rightarrow e^+ e^- \]

KK excitations for different model parameters

Drell-Yan

\[ \frac{d\sigma}{dM} \text{ (pb}/\text{GeV}) \]

\[ M_{ll} \text{ (GeV)} \]

10^{-6} - 10^{-2}
$f\bar{f} \rightarrow f'\bar{f}'$ on a graviton resonance

$1/N \, dN/dz$

$z$

$\cos \Theta$

Easy to ID
So in RS, $\langle R \rangle = -20 k^2$ thus

$$\frac{\langle R^2 \rangle}{M_*^2} = -20 \frac{k^2}{M_*^2} \quad \text{w/} \quad k^2 \leq M_*^2$$

$\therefore$ potentially large corrections from $L_2^*$

$\sim \alpha k^2 / M_*^2$

[Kim, Kyee & Lee '00]

*recall, in D=5 only $L_2$ is non-zero!*
Some differences ... similar to graviton brane kinetic terms

\[ M_{pl} = \frac{M_{*}^3}{k} \left( 1 + 4\alpha \frac{k^2}{M_{*}^2} \right) \]

\[ J_1(x_n) + \frac{4\alpha k^2/M_{*}^2}{1 - 4\alpha k^2/M_{*}^2} \quad J_2(x_n) = 0 \]

- KK mass shifts

\[ J_1 + \lambda J_1 \]

Bessel functions

\[ \Omega \]

\[ \mathcal{L} = -\frac{1}{\Lambda^2} \sum \left[ \frac{1 + 2\Omega}{1 + 2\Omega + \Omega^2 x_n^2} \right]^{1/2} h_{mn}^{(n)} T^{mn} \]

- KK coupling shifts

- \( \alpha < 0 \) [no tachyons]
  - [Charmousis + Du-faux '04]
  - [Brax, Chatillon + Steer '04]

- \( -\frac{1}{2} \leq \Omega \leq 0 \), \( \Omega = 0 = \text{usual RS model} \)

- couplings are KK level dependent!

  and vanishing for \( \Omega = -\frac{1}{2} \)!

- level shifts (weak)

- Can we measure / constrain \( \Omega ?? \)

Use \( m_2/m_1 = P_2/P_1 \) ratios : \( \Omega \) to \( \pm 0.01 \)?
Root shifts due to $L_2$ in RS

$\Omega$

Graviton masses

$\mu_{kk} = \gamma_{kk} e^{-k \mu}$
Graviton Coupling Strengths

Coupling in units of $1/\Lambda_{\pi}$

$\omega$

Vanish
$e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}$

$\sigma$ (fb)

$\sqrt{s}$ (GeV)

$1^{\text{st}}$ graviton resonance

$\Omega$

$2^{\text{nd}}$

$3^{\text{rd}}$

$K.K$ spectrum shifted + resonances narrowed
KK's are getting narrow quite quickly.

Higher KK's get narrow more quickly...

$k/n_{pl} = 0.03$
KK mass ratios

$\frac{R_2}{R_1} \Rightarrow \Omega$ measured at the level 0.01 or better!

determined at $\approx 1\%$ level

graviton
Summary

- The presence of higher curvature terms in the action for gravity can lead to visible modifications in our favorite Extra Dim theories...
  
  - KK Spectrum + coupling shifts in RS
  - New features in BH production / properties in ADD (thresholds, stability)

- More work needs to be done to elucidate these exciting possibilities... and to find other potential signatures