Physics at
e^+e^- Linear Colliders

4. Supersymmetric particles

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In this final lecture, I would like to discuss supersymmetry at the LC. Supersymmetry is not a part of the Standard Model. However, it is a canonical example for the discussion of exotic particle searches and new physics.

Among theories of new physics at the TeV energy scale, supersymmetry (SUSY) is strongly motivated:

SUSY allows the Higgs boson mass to be calculable and gives a mechanism for electroweak symmetry breaking.

SUSY predicts the correct unification relation among the SM gauge couplings.

SUSY gives a natural explanation for cosmological dark matter.

SUSY is compatible with small precision EW corrections and a light Higgs boson.
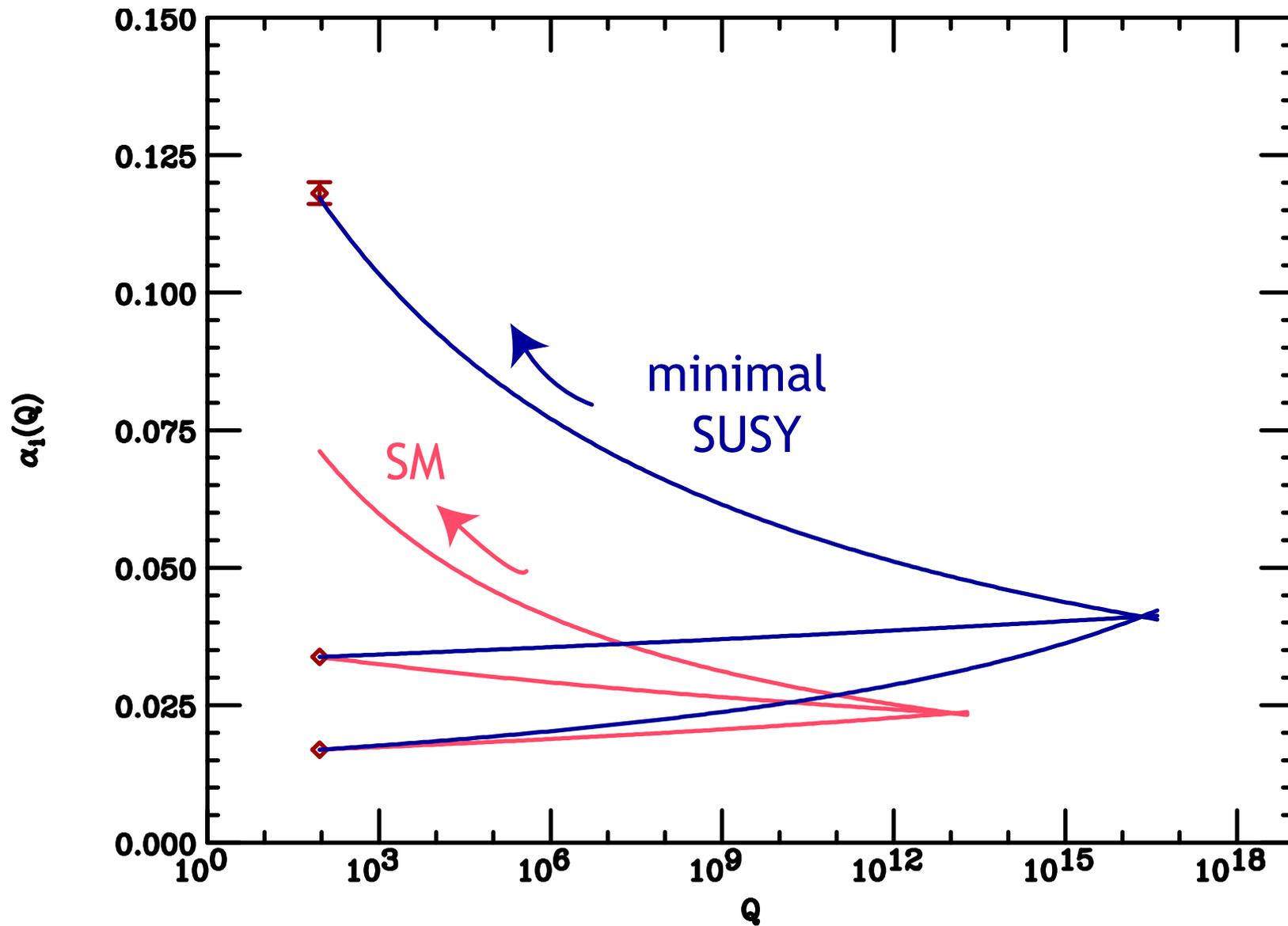
SUSY is also interesting for Linear Collider studies because it is a weakly-coupled theory with complex and rich phenomenology.

SUSY then gives a laboratory in which to illustrate the capabilities of the LC to analyze the properties of exotic particles.

The most important property of SUSY is that ...

it is not possible to build a model in which only part of Nature is supersymmetric.

Let Q be a supersymmetry:

$$Q\,|b\rangle = |f\rangle \qquad Q\,|f\rangle = |b\rangle \qquad [H, Q] = 0$$

$\{Q, Q^\dagger\}$ is nonzero: $\langle a|\,\{Q, Q^\dagger\}\,|a\rangle = \|Q\,|a\rangle\|^2 + \|Q^\dagger\,|a\rangle\|^2$

conserved: $[H, \{Q, Q^\dagger\}] = 0$

a Lorentz vector: $\{Q_a, Q_b^\dagger\} = \gamma_{ab}^\mu R_\mu$

Coleman-Mandula: $R_\mu = P_\mu$

so every particle in Nature has a partner:

$$\gamma \rightarrow \tilde{\gamma} \qquad W^+ \rightarrow \tilde{w}^+$$

$$e_L^- \rightarrow \tilde{e}^- \qquad e_R^- \rightarrow \tilde{\tilde{e}}^-$$

even $\qquad G_{\mu\nu} \rightarrow \Psi_{\mu a}$

SUSY must be spontaneously broken;
otherwise e⁻ and ẽ⁻ would be degenerate.

tree-level SUSY breaking implies

$$\sum_i m_{fi}^2 - \sum_i m_{bi}^2 = 0$$

this also holds for each sector of SM quantum numbers.

so, SUSY must be broken by a sector outside the SM,
coupling to the SM particles by loop effects or gravity

then, e.g.,

$$m_b^2 \sim \frac{\Lambda^4}{m_{Pl}^2}$$

The effective Lagrangian of SUSY-breaking is thus a window into physics at very short distances.

The goal of an experimental program is SUSY is to measure the parameters of this effective Lagrangian precisely, to search for a pattern that will reveal the structure hidden there.

Couplings in SUSY:

$$L = L_{kinetic} + L_W + L_{soft}$$

ingredients:

chiral supermultiplet:
   complex boson ('sfermion') + left-handed fermion:

(ϕ_f, ψ_f)

$$\nu = (\tilde{\nu}, \nu) \qquad I^3 = +\frac{1}{2}, Y = -\frac{1}{2}$$

$$e^- = (\tilde{e}^-, e_L^-) \qquad I^3 = -\frac{1}{2}, Y = -\frac{1}{2}$$

$$\overline{e}^+ = (\tilde{e}^+, e_L^+) \qquad I^3 = 0, Y = +1$$

gauge supermultiplet:
   gauge boson + left-handed fermion ('gaugino'):

(A_μ^a, λ_L^a)

$$L_{kin} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \overline{\lambda}^a i\gamma \cdot D\lambda^a$$
$$+ \overline{\psi}_f i\gamma \cdot D\psi_f + D_\mu \phi_f^* D^\mu \phi_f$$
$$+ \sqrt{2} g(\phi_f^* \lambda^a t^a \psi_f + h.c.) + \frac{g^2}{2}(\phi_f^* t^a \phi_f)^2$$

this is determined only by the SM gauge couplings

$$L_W = -\left|\frac{\partial W}{\partial \phi_f}\right|^2 + \psi_{f1}\psi_{f2}\frac{d^2 W}{\partial \phi_{f1}\partial \phi_{f2}}$$

with $\quad W = y_e \overline{e} H_1 L + y_d \overline{d} H_1 Q - y_u \overline{u} H_2 Q - \mu H_1 H_2$

this involves the Yukawa couplings

and one new parameter $\quad \mu$

Two Higgs fields, H$_1$ and H$_2$, are needed: $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$

SUSY breaking is parametrized by the effective Lagrangian:

$$L_{soft} = -M_f^2|\phi_f|^2 - \frac{1}{2}m_a\lambda^a\lambda^a + h.c.$$
$$-A_e y_e \bar{e} H_1 L - A_d y_d \bar{d} H_1 Q + A_u y_u \bar{u} H_2 Q$$
$$+B\mu H_1 H_2 + h.c.$$

Each coefficient here could actually be a matrix in flavor.

If masses are generated at a high mass scale M, RG effects
can be important in determining the observed spectrum:

initial SUSY parameters
at M
$\longrightarrow$
observed SUSY parameters
at TeV scale
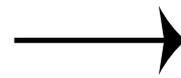
gauginos:

$$m_i(m_Z) = \frac{\alpha_i(m_Z)}{\alpha_i(M)} m_i(M) , \qquad i = 1, 2, 3$$

'gaugino unification': $\quad m_1 \ : \ m_2 \ : \ m_3 \ = \ 0.5 \ : \ 1.0 \ : \ 3.5$

sfermions:

$$m_f^2(m_Z) = m_f^2(M) + \sum_i \frac{2}{b_i} C_2(r_i)(\alpha_i^2(m_Z) - \alpha_i^2(M)) \frac{m_{\tilde{2}}^2}{\alpha_2^2}$$

compare the initial condition at M in 'gauge mediated
SUSY breaking':

$$m_f^2 = \sum_i 2C_2(r_i)\alpha_i^2(M) \frac{m_{\tilde{2}}^2}{\alpha_2^2}$$

Here is a spectrum generated from universal initial conditions at the Planck scale:
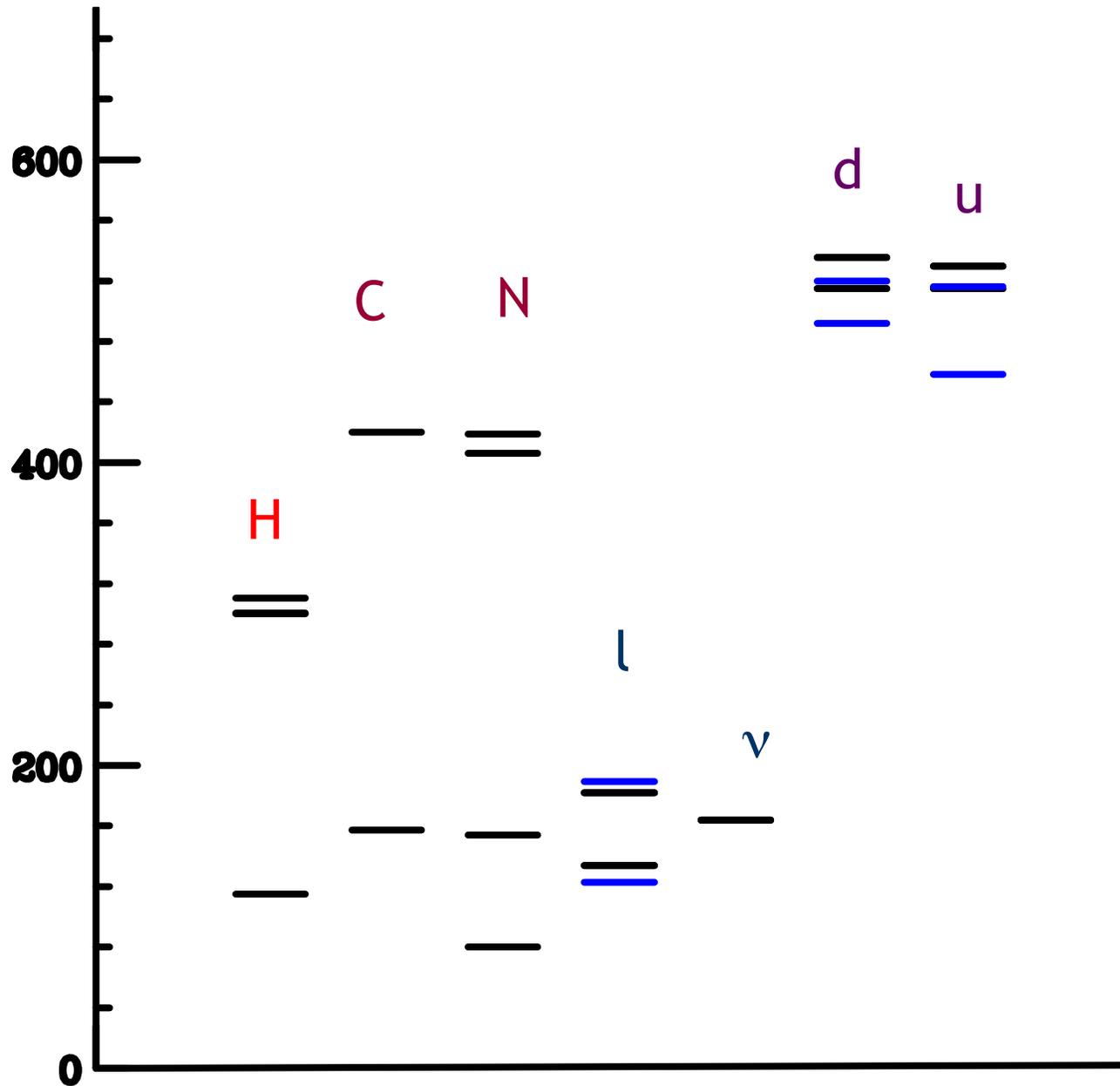
Here is a spectrum generated from boundary conditions
with less universality:
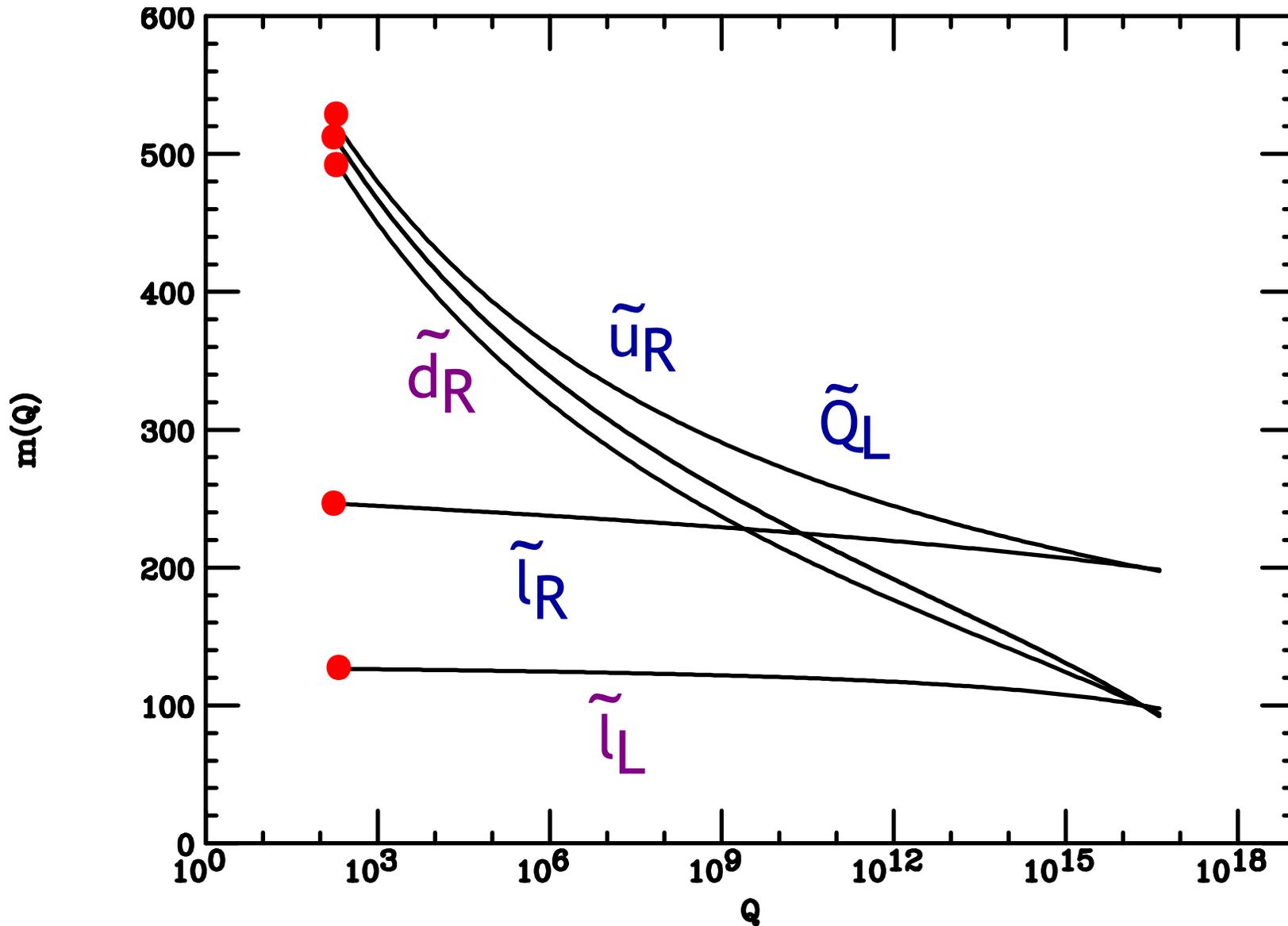
In SUSY, the Higgs boson masses obey ordinary RG equations; the additive contribution to m^2(H) discussed yesterday cancels between boson and fermion loop diagrams.

Further, looking into the RG evolution of the $\tilde{t}$ and H$_2$ masses in more detail, we find a mechanism for electroweak symmetry breaking.

$$\frac{d}{d\log Q}M_2^2 = \frac{1}{4\pi^2}\left[3y_t^2(M_2^2 + M_{\tilde{t}}^2 + M_{\tilde{t}}^2 + A_t^2) + \cdots\right]$$

$$\frac{d}{d\log Q}M_{\tilde{t}}^2 = \frac{1}{4\pi^2}\left[2y_t^2(M_2^2 + M_{\tilde{t}}^2 + M_{\tilde{t}}^2 + A_t^2) - \frac{32}{3}g_s^2 m_3^2 + \cdots\right]$$

$$\frac{d}{d\log Q}M_2^2 = \frac{1}{4\pi^2}\left[y_t^2(M_2^2 + M_{\tilde{t}}^2 + M_{\tilde{t}}^2 + A_t^2) - \frac{32}{3}g_s^2 m_3^2 + \cdots\right]$$

As a byproduct, the $\tilde{t}$'s can also be left significantly lighter than the other squarks.

Several groups of the new states mix among themselves.

For sfermions, the L-R mixing is sensitive to the
supersymmetric Higgs parameters

$$
\begin{pmatrix}
M_{\tilde{e}}^2 + m_e^2 & m_e(A_e - \mu \tan \beta) \\
m_e(A_e - \mu \tan \beta) & M_e^2 + m_e^2
\end{pmatrix}
\qquad
\begin{pmatrix}
\tilde{\bar{e}}^* \\
\tilde{e}
\end{pmatrix}
$$

For the $\tilde{b}$, $\tilde{w}$, and $\tilde{h}$ fermions, the mixing involves $\tan\beta$ and weak interaction parameters:

$$
\begin{pmatrix} \widetilde{w}^- & \tilde{h}_1^- \end{pmatrix}
\begin{pmatrix} m_2 & \sqrt{2}s_\beta m_W \\ \sqrt{2}c_\beta m_W & \mu \end{pmatrix}
\begin{pmatrix} \widetilde{w}^+ \\ \tilde{h}_2^+ \end{pmatrix}
$$

$$
\begin{pmatrix}
m_1 & 0 & -c_\beta s_w m_Z & s_\beta s_w m_Z \\
0 & m_2 & c_\beta c_w m_Z & -s_\beta c_w m_Z \\
-c_\beta s_w m_Z & c_\beta c_w m_Z & 0 & -\mu \\
s_\beta s_w m_Z & -s_\beta c_w m_Z & -\mu & 0
\end{pmatrix}
\begin{pmatrix}
\tilde{b}^0 \\
\widetilde{w}^0 \\
\tilde{h}_1^0 \\
\tilde{h}_2^0
\end{pmatrix}
$$

In addition, we do not know the relative size of m_1, m_2, and μ. The lightest mass eigenstates could be gaugino-like or higgsino-like.

Call the mass eigenstates 'charginos' $\tilde{C}_i$ and 'neutralinos' $\tilde{N}_i$.
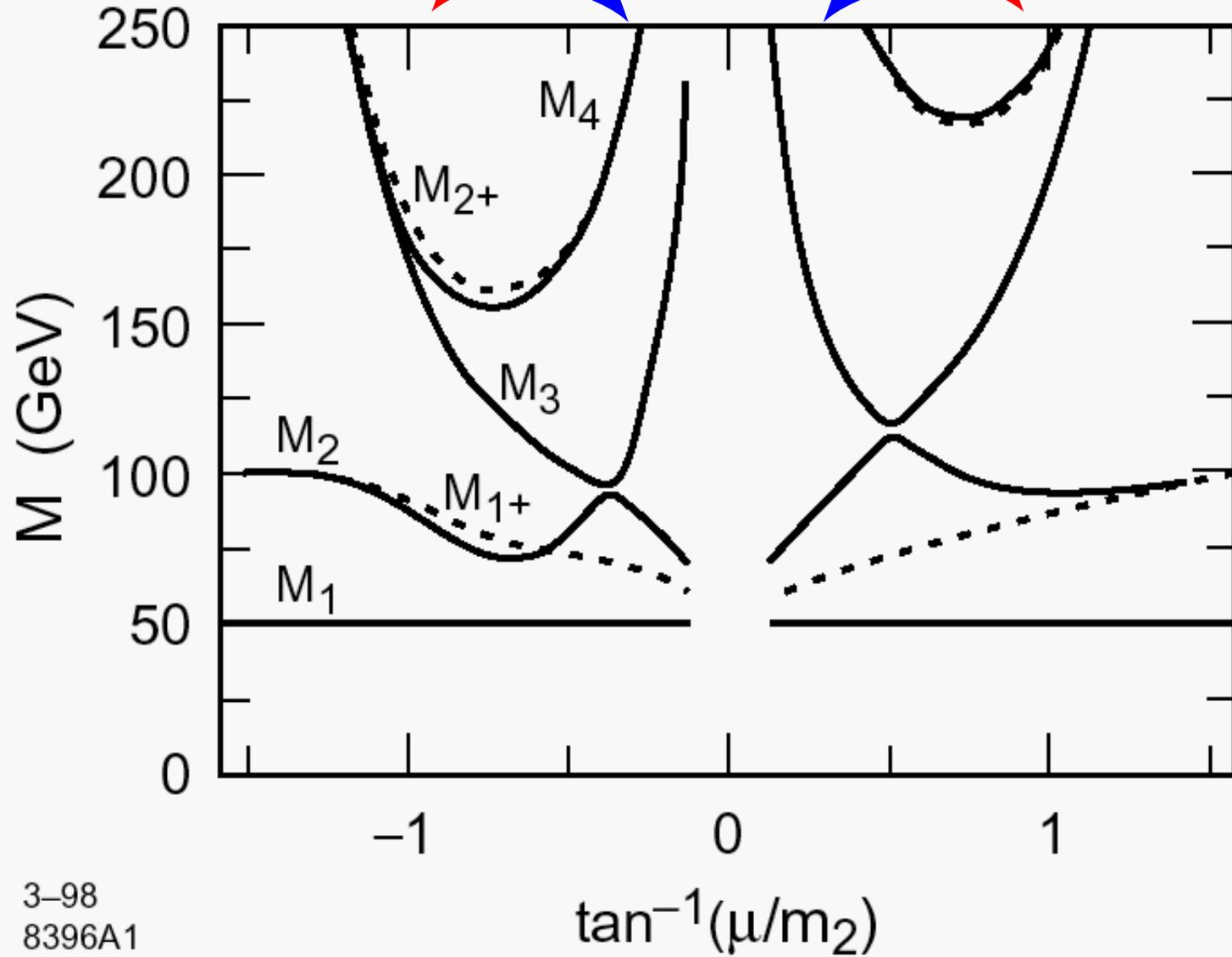
So what are the important parameters of supersymmetry that we must measure ?

mechanism of SUSY breaking:

$$m_1 \ : \ m_2 \ : \ m_3$$

$$M(\tilde{\bar{e}}) \ : \ M(\tilde{e}) \ : \ M(\tilde{q})$$

mechanism of electroweak symmetry breaking:

$$A_f \ , \ \ \mu \ , \ \tan \beta$$

SUSY breakings and flavor:

$$M(\tilde{e}) : \ M(\tilde{\mu}) \ : \ M(\tilde{\tau})$$

When will SUSY be found?

If SUSY is to control the radiative corrections to the Higgs mass and VEV, SUSY must appear not far above the weak interaction scale.

Quantitative estimates ('naturalness limits') give roughly

$$m_2 \;<\; 200 \text{ GeV}, \qquad m_3 \;<\; 700 \text{ GeV}$$

Thus, SUSY must be seen at the LHC.

Hopefully, it willl also appear at the Tevatron.

The LHC will measure some SUSY particle masses precisely

e.g. $M(\tilde{\bar{q}})$, $m(\tilde{N}_2) - m(\tilde{N}_1)$

However, at the LHC

SUSY is produced in cascades starting from $\tilde{q}$, $\tilde{g}$
typical events are complex
the major background to SUSY is SUSY

This makes it difficult to interpret experimental results in a
model-independent way.

The strength of the LC program on SUSY is that the events are simple.

Using our understanding of the SM, we can analyze these events and obtain model-independent information about the SUSY parameters.

There is much to be gained from the detailed study of just the lightest states of the SUSY spectrum,

$$\tilde{\bar{e}} \qquad \tilde{e} \qquad \tilde{N}_1 \qquad \tilde{N}_2 \qquad \tilde{C}_1$$

The simplest SUSY process at the LC is $e^+e^- \to \tilde{\mu}\,\tilde{\mu}^*$.

Typically $\quad \tilde{\mu} \to \mu + \tilde{N}_1$, with $\tilde{N}_1$ unobserved.

This process is distinguishable from $\gamma\gamma \to \mu\mu$ background by the large missing p_t; background from W^+W^- can be understood or removed using beam polarization.

For e⁺e⁻ annihilation to a massive scalar pair,

$$\mathcal{M} = e^2 \beta \sin\theta \cdot f_{IJ} \qquad \beta = \left(1 - \frac{4m^2}{s}\right)^{1/2}$$

where f_{IJ} are the γ - Z interference amplitudes discussed in the first lecture.   The cross section depends on SM quantum numbers in a completely model-independent way.

Recall that the $|f_{IJ}|^2$ give large polarization asymmetries

$e^-_R \rightarrow \tilde{\bar{\mu}}$ 1.69 $e^-_L \rightarrow \tilde{\bar{\mu}}$ 0.42

$e^-_R \rightarrow \tilde{\mu}$ 0.42 $e^-_L \rightarrow \tilde{\mu}$ 1.98

The characteristic shape of the cross section is

$$\frac{d\sigma}{d\cos\theta} \sim \beta^3 \sin^2\theta$$

Since $\tilde{\mu}$ is a scalar, it decays isotropically to μ .

After a boost, this gives a flat energy distribution for the μ.
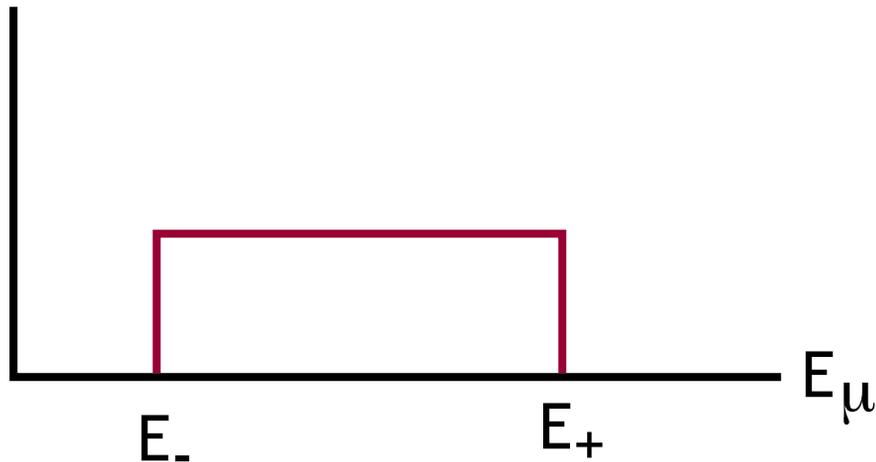
$$E_- = \gamma(1-\beta)E_N \qquad E_+ = \gamma(1+\beta)E_N$$

where $\quad E_N = \dfrac{m_{\tilde{\mu}}^2 - m_N^2}{2m_{\tilde{\mu}}}$

These equations can be solved algebraically for $M(\tilde{\mu})$, $m(\tilde{N}_1)$.

Blair and Martyn

$\tilde{\nu}_e\,\tilde{\nu}_e^*$

(b)

Theory
Fit+BG

Data

Events/Bin

E_e (GeV)

5–96

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Baer et al.
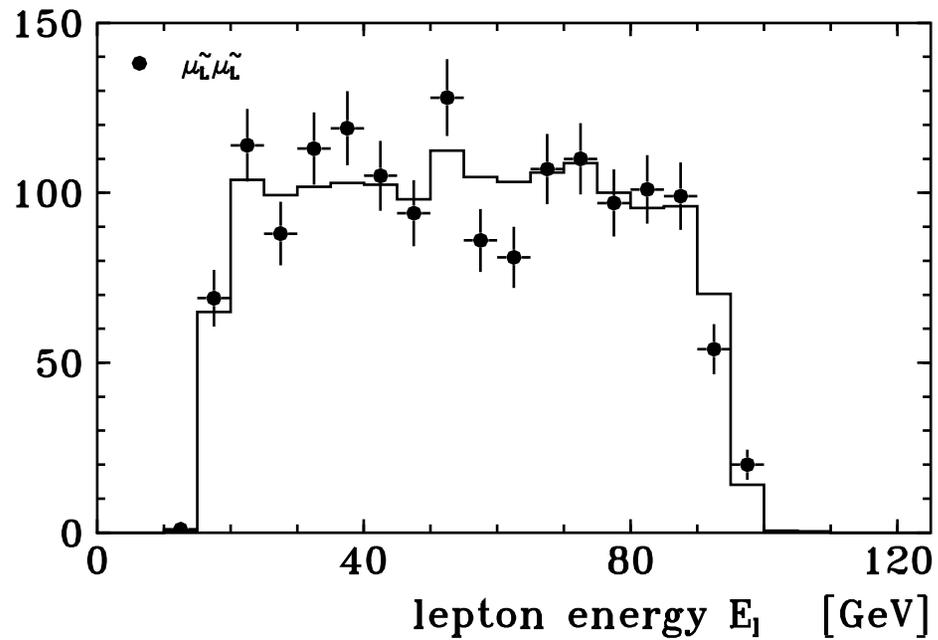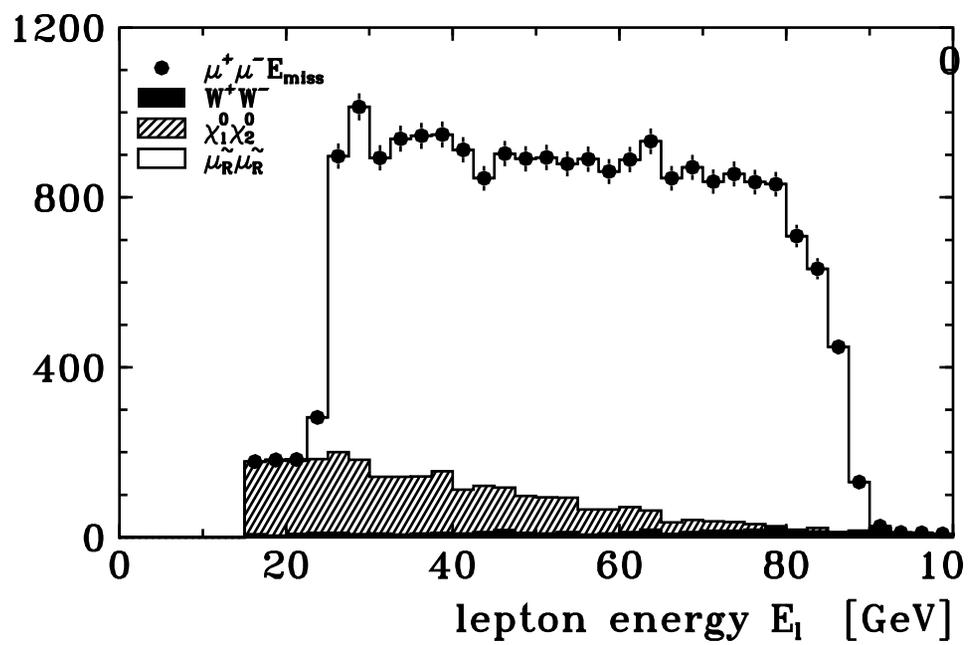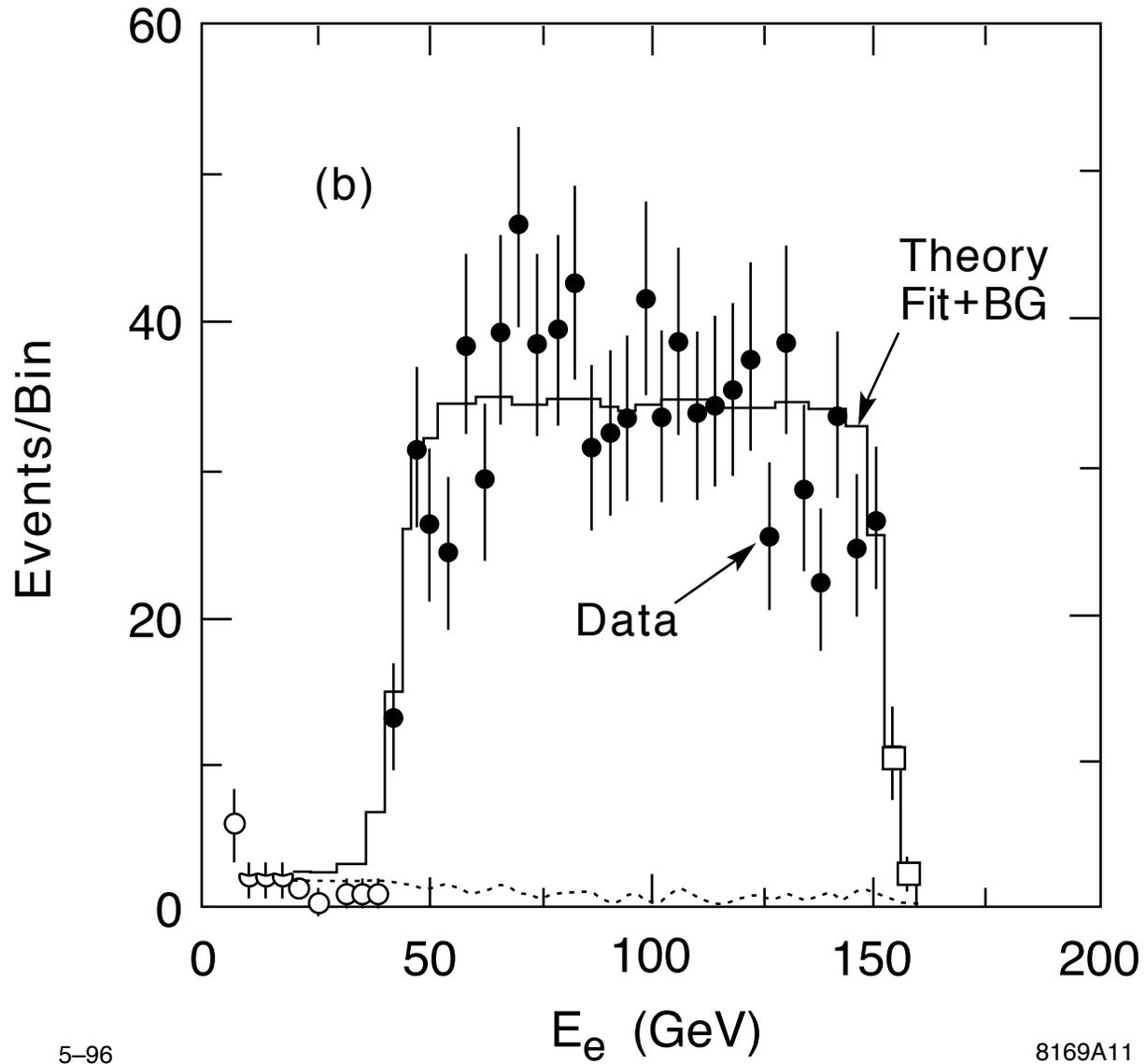
The process $e^+e^- \rightarrow \tilde{\tau}\tilde{\tau}^*$ introduces the complication of $\tilde{\tau}$ mixing.   This generalizes the amplitude formulae to, for example

$$\mathcal{M}(e^-_R e^+_L \rightarrow \tilde{\tau}^-_1 \tilde{\tau}^+_1) = e^2 \beta \sin\theta \left[\cos^2\theta_\tau f_{RR} + \sin^2\theta_\tau f_{RL}\right]$$

Measuring the cross section from polarized beams, we obtain the mixing angle, and, as a cross-check, $M(\tilde{\tau})$.
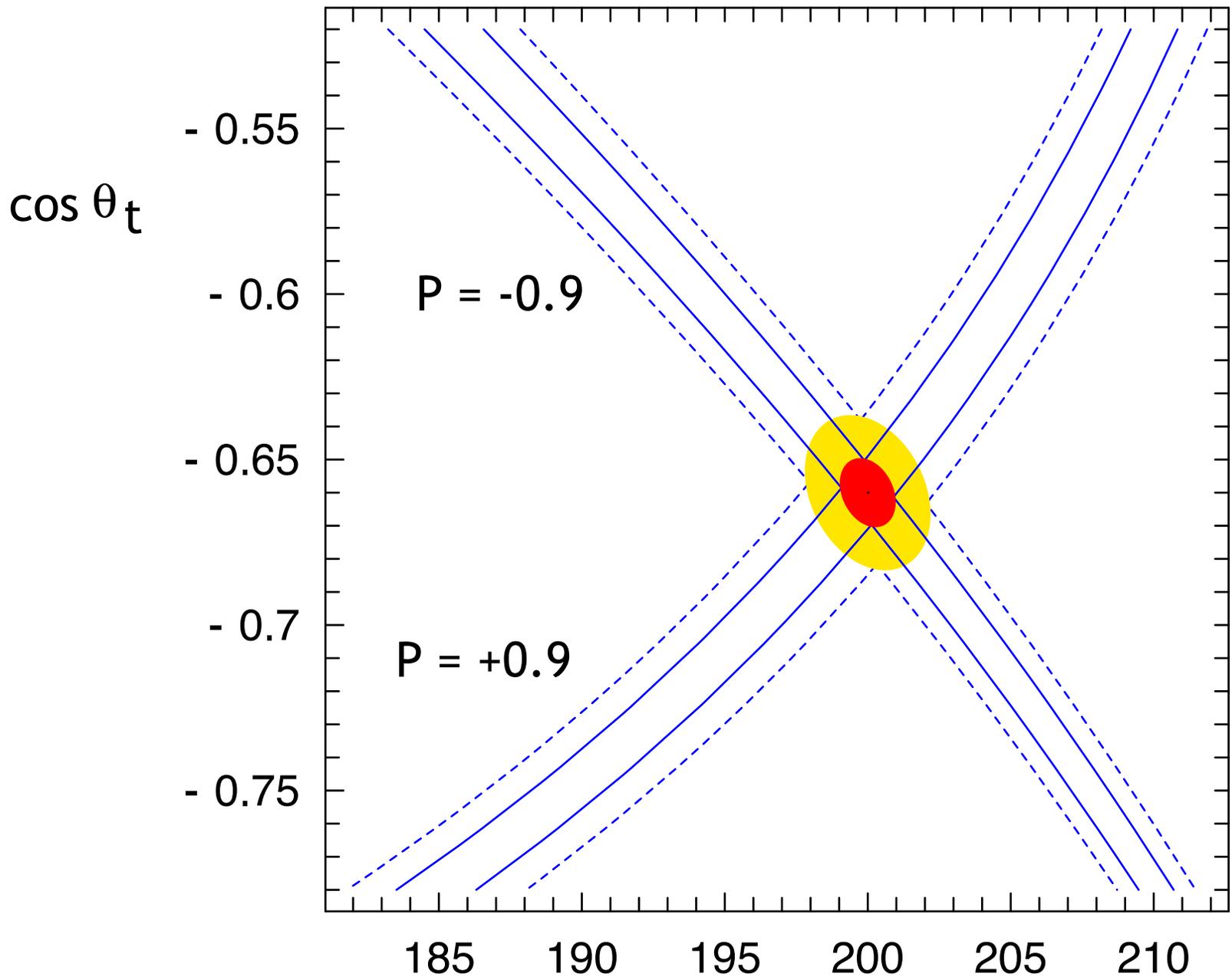
The process $e^+e^- \to \tilde{e}\,\tilde{e}^*$ brings in another diagram involving $\tilde{N}$ exchange:
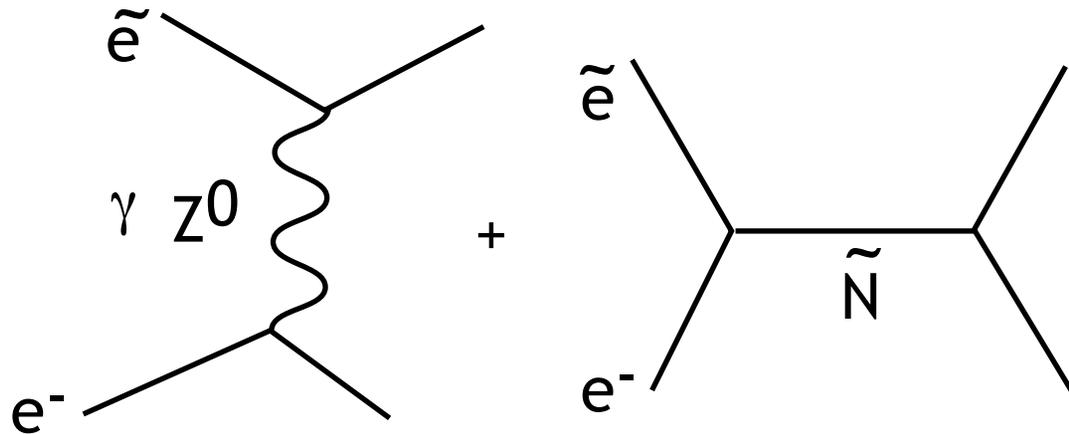
For example, ignoring neutralino mixing,

$$\mathcal{M}(e_R^- e_L^+ \to \tilde{e}^- \tilde{e}^+) = e^2 \beta \sin\theta \left[f_{RR} - \frac{1}{c_w^2} \frac{s}{m_{\tilde{b}}^2 - t} \right]$$

It is typical that the t-channel $\tilde{N}$ exchange is actually the dominant contribution.

The $\tilde{N}$ exchange process has a number of possible applications:

It can be used to determine the gaugino-higgsino mixing parameters.

If the mixing is constrained from another process, it can be used to measure the vertex

$$
\tilde{e} \diagdown \!\!\!\diagup \tilde{b} \\
e^{-} \diagup
$$
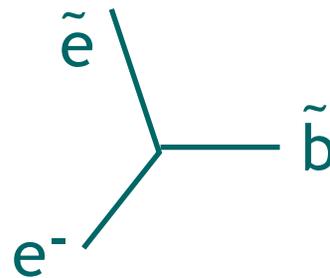
and test the symmetry relations of SUSY. Accuracies better than 1% are possible.

It induces the process $e^{-}e^{-} \rightarrow \tilde{e}\,\tilde{e}$. This process is S-wave, so $\sigma \sim \beta$. This allows a very accurate (0.1%) threshold mass measurement.

$e^- e^- \rightarrow \widetilde{\widetilde{e}}^- \widetilde{\widetilde{e}}^-$

variation of $m(\widetilde{\widetilde{e}})$ by 100 MeV, for $m(\widetilde{\widetilde{e}}) = 150$ GeV
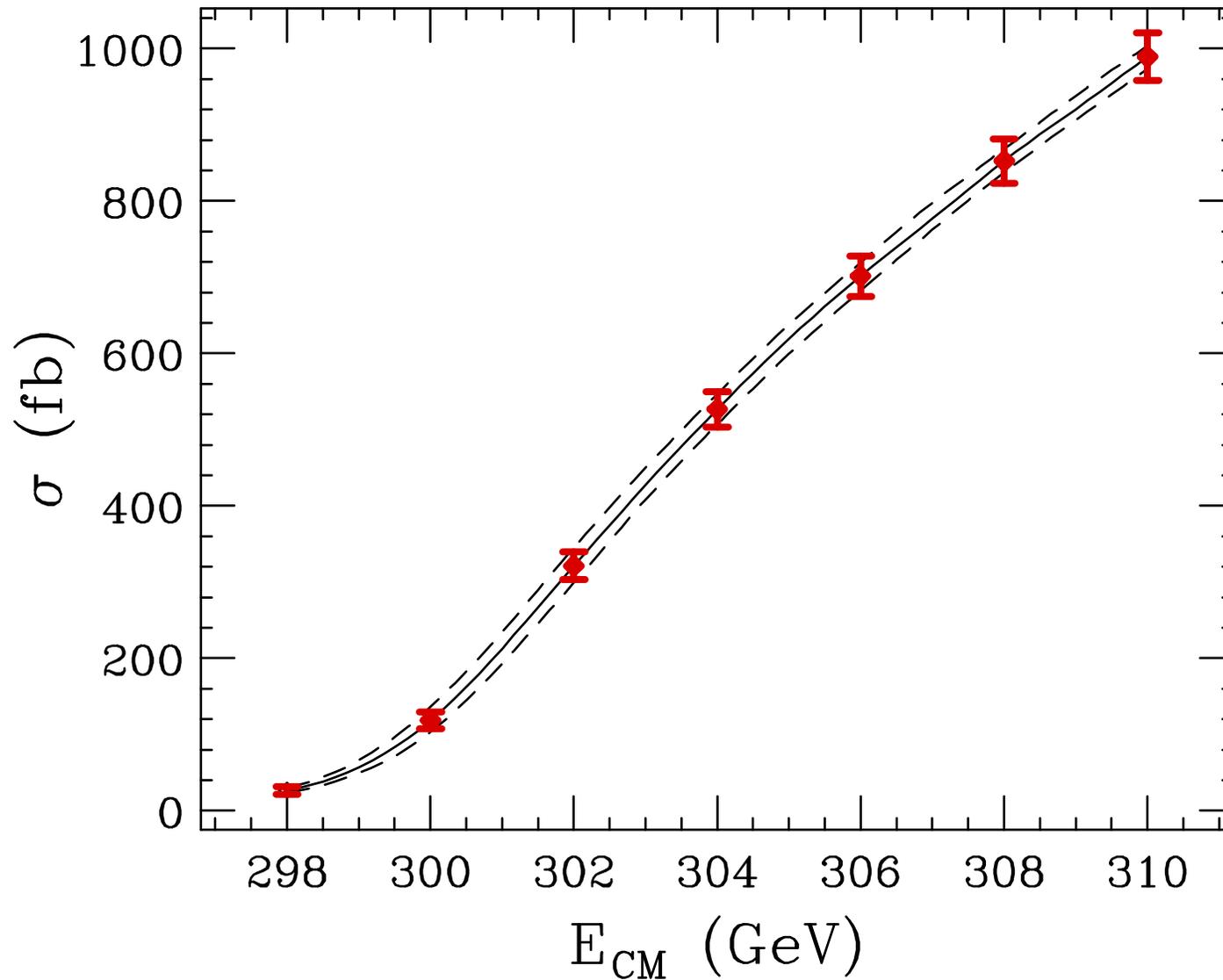
Feng et al.

Since different mechanisms for SUSY breaking may or may not be universal over fermion generations, it is important to test as accurately as posssible whether the sleptons of e, μ, τ are degenerate. For $\tilde{e}$, $\tilde{\mu}$, part-per-mil accuracy could be achieved by comparing kinematic endpoints. For $\tilde{\tau}$, this comparison is more of a challenge.

The process $e+e- \rightarrow \tilde{C}^- \tilde{C}^+$ brings in all of the complications we have seen in the various slepton systems. There are competing s- and t-channel contributions

$$\tilde{C}^- \quad \gamma \quad Z^0 \quad e^- \quad + \quad \tilde{C}^- \quad \tilde{\nu} \quad e^-$$
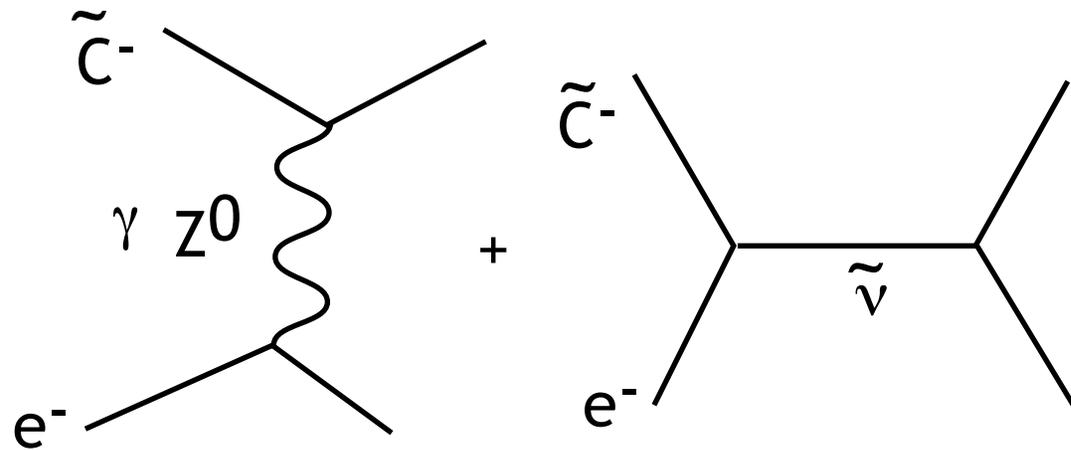
and a nontrivial mixing problem. Though it is not so transparent, precision measurements of the cross section can sort out these issues.

This is especially important, because the $\tilde{C}^-$ is the SUSY state most likely to appear at a 500 GeV e^+e^- collider.

$\tilde{C}_1^+ \tilde{C}_1^-$

di-jet energy E_{jj} [GeV]

di-jet mass m_{jj} [GeV]
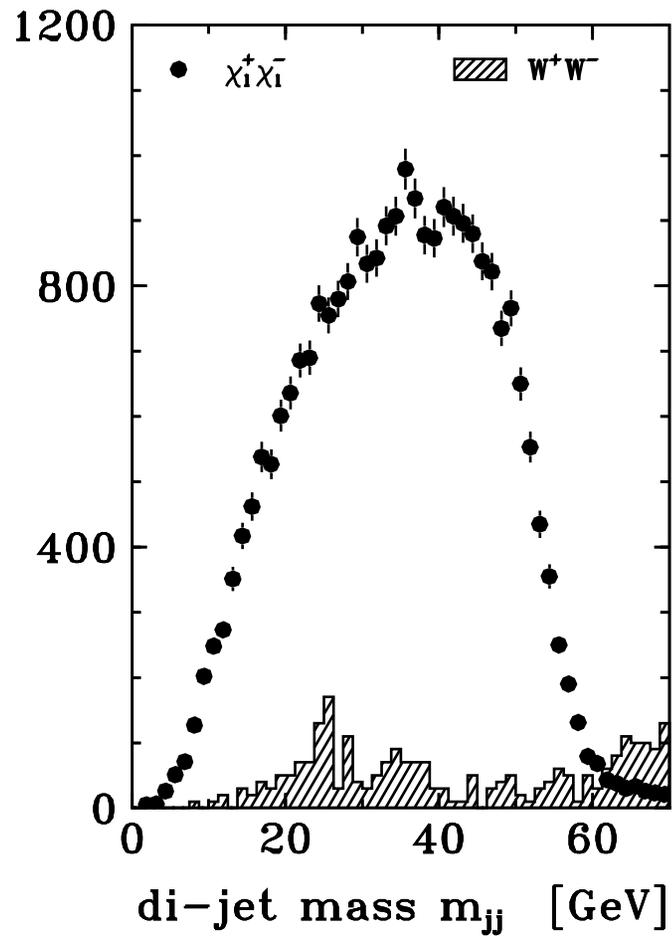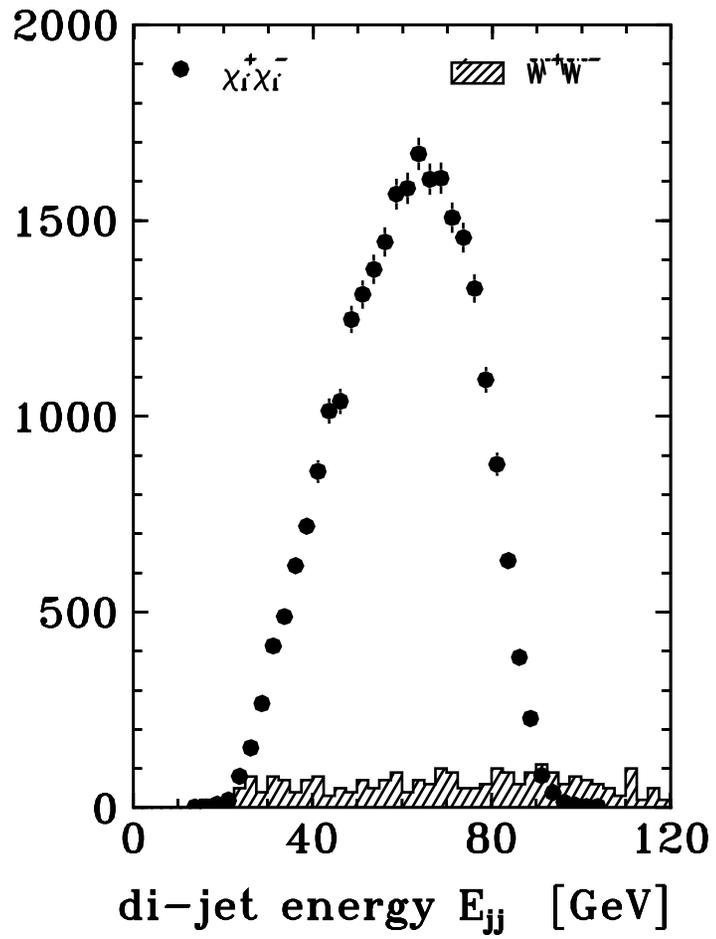
Blair and
Martyn

Here is one simple example of the $\tilde{C}^-$ physics. Consider producing $\tilde{C}^- \tilde{C}^+$ with an e^-_R beam at high energy. The e^-_R does not couple to the $\tilde{\nu}$, so the process simplifies to

$\tilde{C}^-$

B

e^-_R
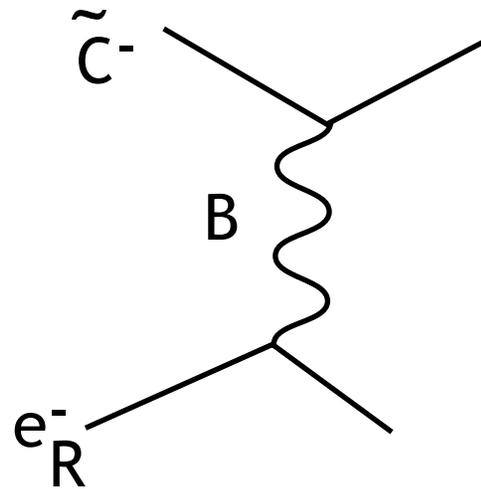
The w has Y=0, so the B couples only to the higgsino component of $\tilde{C}$. The forward and backward production measure separately the $\tilde{C}^-$ and $\tilde{C}^+$ mixing angles.

$\sigma \, (e^-_R \, e^+ \rightarrow \tilde{C}^+_1 \, \tilde{C}^-_1)$ (fb)
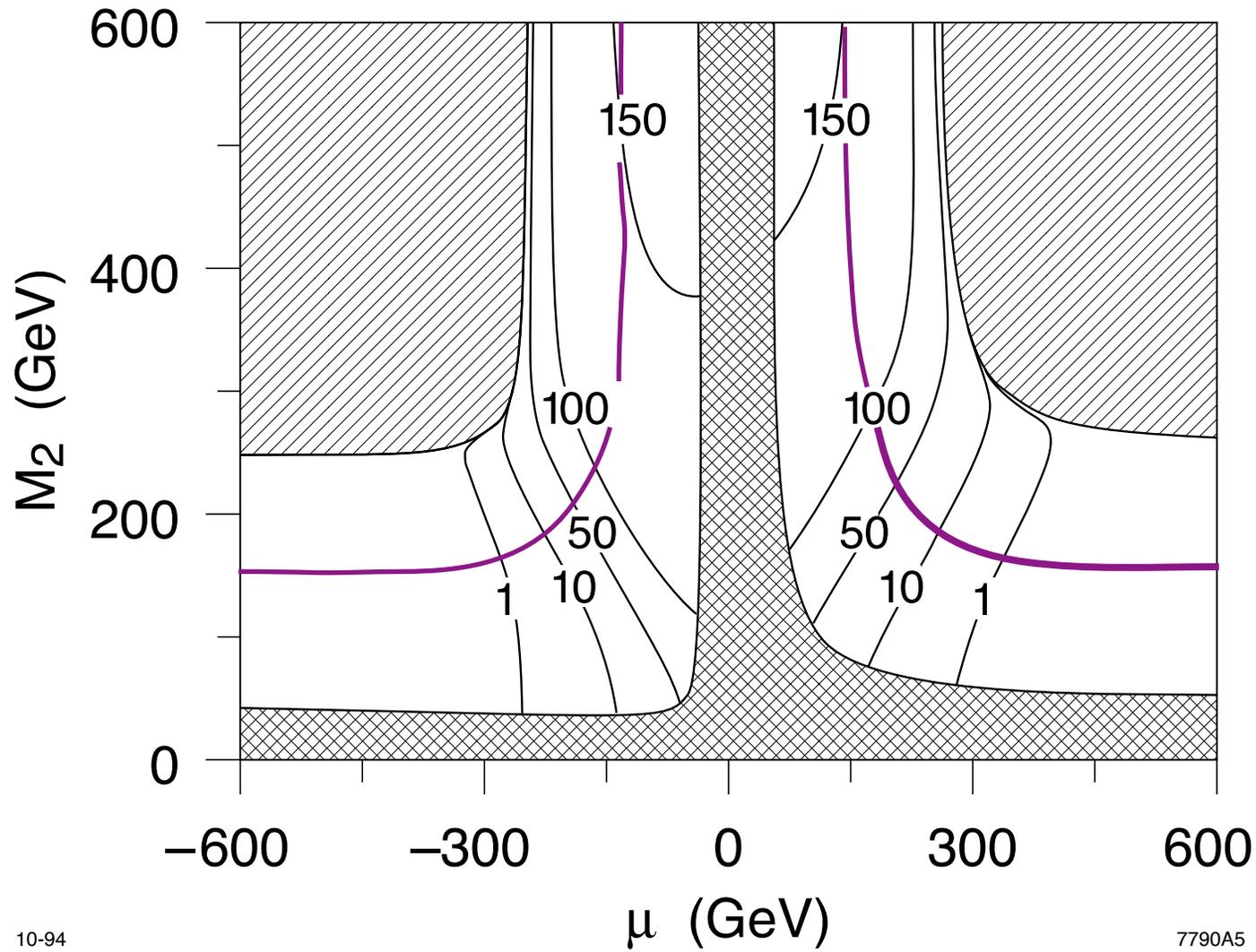
line of fixed $\tilde{C}_1$ mass

Feng et al.

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These examples illustrate the powerful capabilities of the LC to determine the detailed properties and interactions of new particles.

We full expect that the LHC and LC will open a new sector of physics and a new set of fundamental laws to our exploration. Enjoy!