Current status of

$e^+e^- \to t\bar{t}$

in the threshold region

K. Melnikov

SLAC

Plan

1. Early history (1997 - 1998)

2. Turbulent years (1999 - 2000)

3. Renaissance and the future (2001 ÷ ...)

1. Early history

Heavy top quark decays **before** hadronization

(Fadin & Khoze , 1987)

$m_t \sim 170$ GeV ; $t \to Wb$; $\Gamma_t \sim 1.5$ GeV

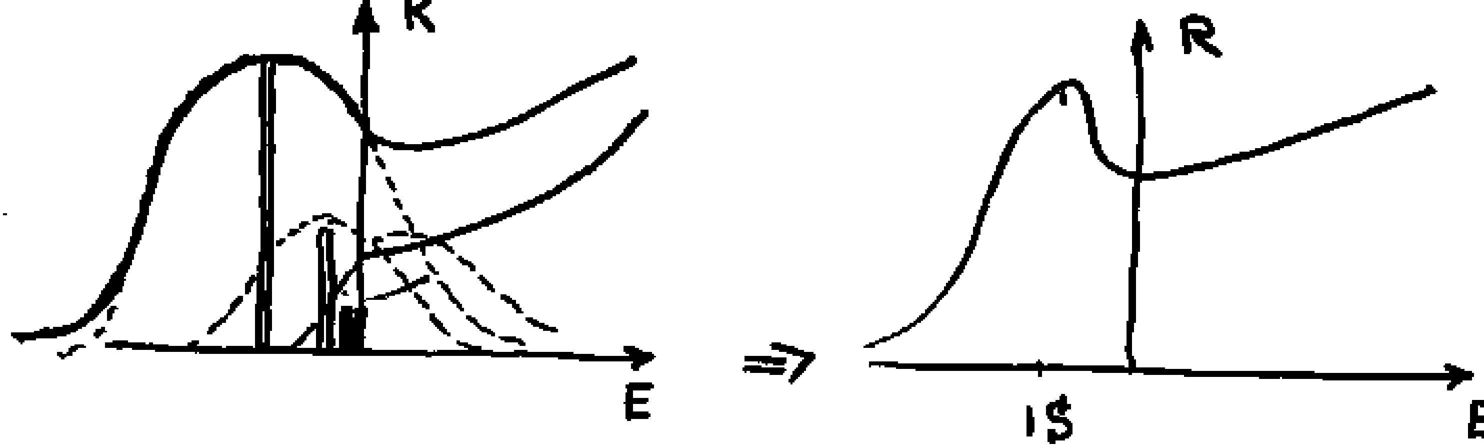
$$\Gamma_t \gg \Lambda_{QCD}$$

$\Rightarrow$ **No** t - hadrons ;

t behaves as a **free** quark.

Important implications for

$t\bar{t}$ threshold in e^+e^-.



* The resonance structure is washed out

* LO approximation: $E = \sqrt{s} - 2m_t$

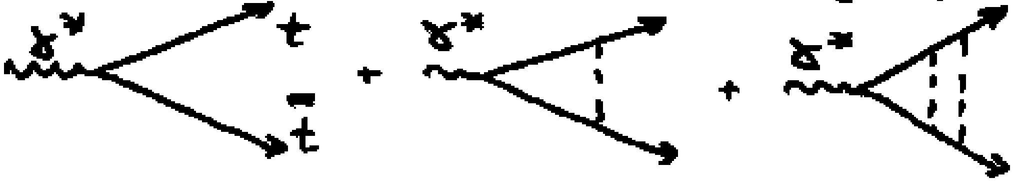
$$\sigma(e^+e^- \to t\bar{t}) \propto \mathrm{Im}\left[\sum_n \frac{\psi_n^*(0)\,\psi_n(0)}{E - E_n + i\Gamma_t}\right]$$

$$|\psi_n(0)|^2 = \frac{(C_F \alpha_s m)^3}{8\pi n^3}, \quad E_n = -\frac{m (C_F \alpha_s)^2}{4n^2}$$

For example : $E_{n=1} \simeq -1.7$ GeV , $E_{n=2} \simeq -0.5$ GeV

* Non-perturbative effects $O\left[\left(\frac{\Lambda_{QCD}}{\Gamma_t^2 m \alpha_s^2}\right)^4\right]$ (Fadin & Yakovlev)

* Comment : in pert. theory LO = infinite sum of Feynman graphs:



$$+ \dots \equiv \sum_{n=0}^{\infty} r_n \left(\frac{\alpha_s}{v}\right)^n$$

It is then relatively clear that the threshold excitation curve is sensitive too:

m_t ÷ position of the peak

α_s ÷ height of the cross section (α_s^3
position of the peak ($m_t \alpha_s^2$)

Γ_t ÷ $\Gamma_t \to \infty$, no peak

Higgs effects ÷ for SM Higgs with $m_H \sim 100$ GeV, the cross-section increases by 10%.

From this list one sees that from threshold excitation curve one can extract various SM parameters.

In 1998, the effort of European NLC community has been summarized in Physics Reports. [299 (1998) p.1-79]

For top threshold, we find there

$$\delta m_t \simeq 120 \text{ MeV}$$

$$\delta \alpha_s \simeq 0.003$$

$$\delta \Gamma_t / \Gamma_t \lesssim 20\%$$

and so everything looked bright.

Then 1999 came ....

4. QCD/top threshold at the NLC

- Top quark is unstable, $\Gamma_t \sim 1.5$ GeV. This implies that:

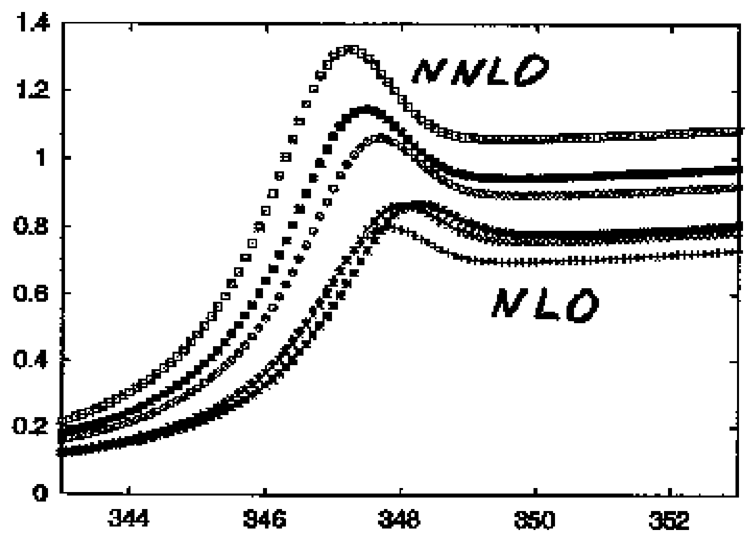
  1. toponium can not be formed;
  2. top threshold cross section is computable from first principles.

- Prospects to measure m_t, Γ_t, α_s, top-Higgs Yukawa coupling were thought to be bright.

- The $\mathcal{O}(\alpha_s^2)$ QCD corrections were not originally computed; apparently it was much too difficult task for a long time. When it was finally done, the result turned out to be disastrous.

$$R = \frac{\sigma_{e^+e^- \to \bar{t}t}}{\sigma_{e^+e^- \to \mu^+\mu^-}}$$

pole mass



$\mu_s = 15, 30, 60$ GeV

$\sqrt{s}$, GeV

Such large corrections were not anticipated and they do mean that the precision physics at the top threshold is rather questionable. Can anything can be done about it?

Two ideas were suggested:

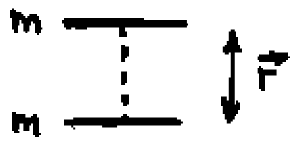
1) Do not use quark <u>pole</u> mass

2) Re-shuffle the series, changing the notion of "leading order".

- Let's look at the position of the peak. Since this is a remnant of the $1S$ toponium, $E_{\rm peak} = 2m + E_{\rm bnd}$. Hence, confronting experimental and theoretical curves in higher orders, we will extract larger and larger values of the pole mass and extrapolating this to an extreme, we conclude that the pole mass does not exist.

- A similar problem has already been encountered in B-physics, and the conclusion was that other masses have to be used.

- To give you an idea of what are we after with these masses:



$$E_{st} = 2m - \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{C_F \alpha_V(\mathbf{q})}{\mathbf{q}^2} e^{i\mathbf{q}\mathbf{r}}.$$

Landau pole in the potential questions the existence of this formula and this can not be right. So let us rewrite it through well defined quantities:

$$E_{st} = 2m(\mu) - \int_{|\mathbf{q}|>\mu} \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{C_F \alpha_V(\mathbf{q})}{\mathbf{q}^2} e^{i\mathbf{q}\mathbf{r}}, \qquad \mu \gg \Lambda_{QCD}$$

The new mass $m(\mu)$ is called the PS (potential subtracted) mass.

It satisfies rather curios "renormalization-group" equation:

$$\frac{d}{d\mu} m(\mu) \sim C_F \alpha_s(\mu),$$

which implies a linear sensitivity to the renormalization scale.

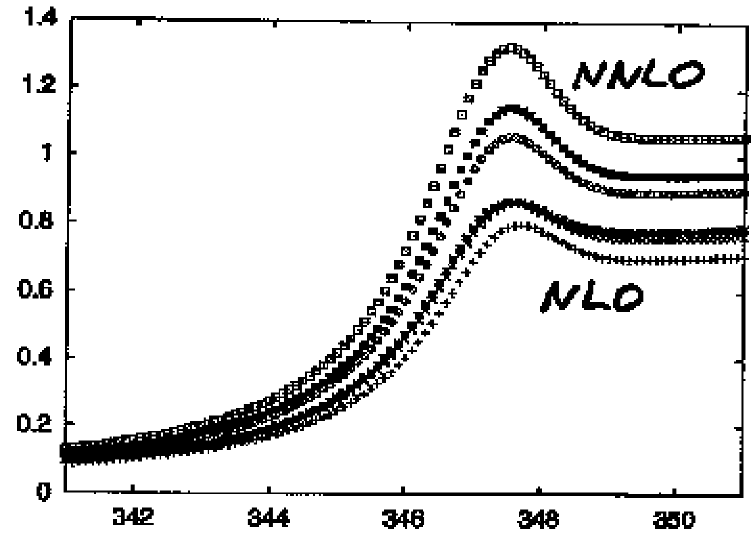
$$m_{pole} = m(\mu)\left\{ 1 + x_1 \frac{\alpha_s}{\pi}\frac{\mu}{m(\mu)} + x_2 \left(\frac{\alpha_s}{\pi}\right)^2 \frac{\mu}{m(\mu)} + \dots \right\}$$

- So, lets fix the threshold mass at some scale μ and recompute the cross section. The value of the pole mass in different orders of PT will differ by a significant amount.

- After that the result looks almost perfect in that the peak position can be determined with very high precision.

- Note, no improvement with the normalization yet.

R, threshold mass



$\mu_s = 15, 30, 60$ GeV

$\sqrt{s}$, GeV

From here: $\delta m_{th} \sim 100$ MeV;

$\delta \overline{m}^{MS}(\overline{m}^{MS}) \simeq 100$ MeV,

- **The height of the cross section at the peak is determined by the wave function of the "toponium".**

$$\sigma_0 = \frac{3\pi}{2m^2}\frac{\Gamma_{T\to e^+e^-}}{\Gamma_t}.$$

$$\Gamma_{T\to e^+e^-} \sim C(\mu)\psi(0)^2, \qquad |\psi(0)|^2 = \frac{(C_F\alpha_s(\mu)m)^3}{8\pi}.$$

- **Sources of corrections:**

  1. **renormalization of the production current;**
  2. **corrections to the Coulomb potential;**
  3. **relativistic corrections.**

- **Corrections to the Coulomb potential are moderate, once the running of the coupling constant is taken into account.**

- $\log\alpha_s$ **corrections are relatively easy to follow:**
  In this approximation, the wave function at the origin reads:

$\Gamma_n \simeq \frac{\pi}{2}^n N_c^n$

$$|\psi(0)|^2 = \left[\psi(0)^2\right]_0 \left\{1 - \left(\frac{\alpha_s}{\pi}\right)\ [5.33] + \left(\frac{\alpha_s}{\pi}\right)^2\ [51.17]\log\left(\frac{1}{\alpha_s}\right) - \left(\frac{\alpha_s}{\pi}\right)^3\ [232.48]\log\left(\frac{1}{\alpha_s}\right)^2\right\}.$$

- **Numerically** $[\alpha_s = 0.15]$**:**

$$|\psi(0)|^2 = \left[|\psi(0)|^2\right]_0 (1 - 0.255 + 0.223 - 0.09)\,,$$

that shows large corrections. How to make sense out of that?

- Note the sign alternating nature of the above corrections.

  Are we missing some guiding principle to organize the series better?

- Re-shuffle the series – summing up $\log \alpha_s$ corrections [A. Hoang, A. Manohar, I. Stewart, T. Teubner].

$$R_{LL} = v \sum_{k,i} \left(\frac{\alpha_s}{v}\right)^k (\alpha_s \log v)^i . \tag{1}$$

  What does it mean?

  At LL – running of the coupling constant.

  At NLL – all the terms in the above equation for $|\psi(0)^2|$ should be counted at once. This gives

$$[0.74/0.97/0.87] \Longrightarrow 0.87, \tag{2}$$

  mild reduction instead of volatile series.

- From known NNLO calculations, one might expect that NNLL calculation with <u>only</u> known terms will not induce large corrections.

- Recent claim: NNLL calculations of the $t\bar{t}$ threshold production cross section give this quantity with the uncertainty of about 2 per cent.

  Opens back the window for precision physics at $t\bar{t}$ threshold(!?)

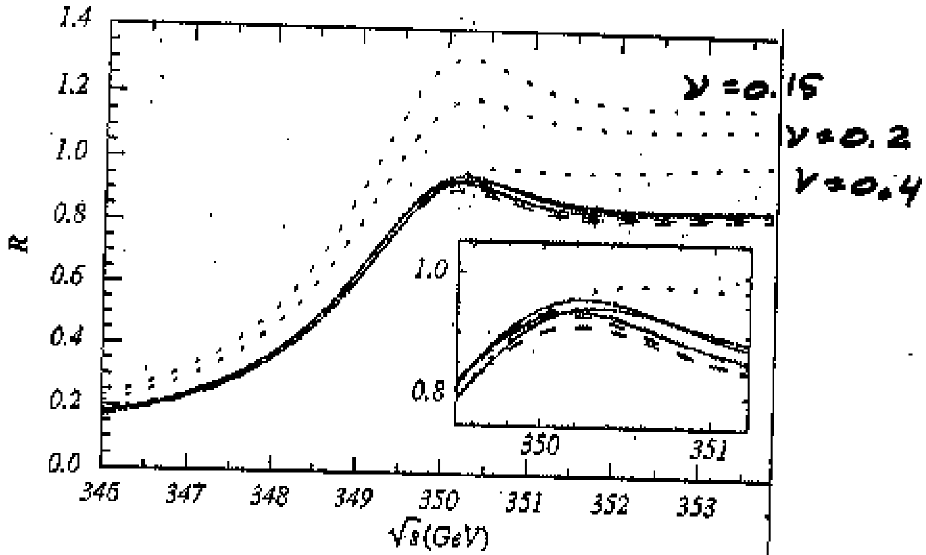
A.H. Hoang, A.V. Manohar, I.W. Stewart and T. Teubner

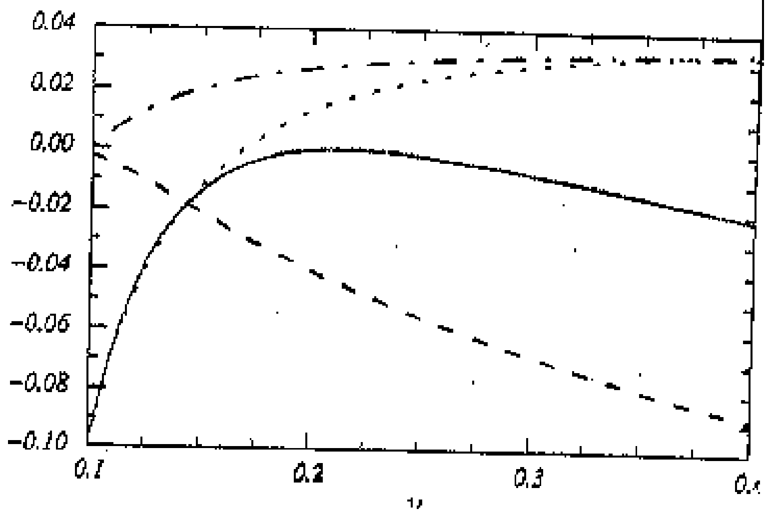
hep-ph/0011254

Based on

$\log(\alpha_s)$ resumm.

$\log(0.15) \simeq -2$





Conclusions

* Top threshold history gives a good example of how stable and reliable theory predictions are...

* Current status:

  $\delta m \sim 100$ MeV

  $\frac{\delta\sigma}{\sigma} \sim 2\%$ [Not universally accepted].

* To settle the matter, we need N^3LO ($O(\alpha_s^3)$) calculation.

  This is extremely difficult undertaking.

* $O(\alpha_s^2)$ corrections to differential distributions have to be studied ($|\vec{p}_t|$, forward-backward asymmetry).

* Improved treatment of the top width.