Limit on the vertical beam size at LC due to crab crossing

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- In ideal case
  $\sigma_y = \sqrt{\frac{\varepsilon_{ny}\beta_y}{\gamma}}$, and can be made arbitrary small by reducing ε_{ny}.
- There is a limit $\overset{\text{on } \sigma_y}{}$ due to radiation in quads (Oide effect).
  $n_\gamma \propto \sqrt{\frac{\varepsilon_n}{\beta\gamma}}\,\gamma$.
  At $\varepsilon_n \to 0$, $n_\gamma \to 0$, and $\sigma_y \to 0$.
  (for there is some opt. β for min σ_y)
- Limit on σ_y due to crab crossing.

Electrons radiate in the effective transverse field $B_\perp = B_s \frac{\alpha_c}{2}$
where B_s – the solenoidal field
α_c – crab crossing angle (between beams)

The number of emitted photons

$$n_\gamma \sim 0.01\,\theta\gamma \sim 0.01\,\frac{L e B_s \alpha_c \gamma}{2E} = \underline{0.005\,\frac{e B_s \alpha_c}{mc^2} L}$$

For $B_s = 4T$, $\alpha_c = 30$ mrad, $L = 4$ m $\Rightarrow$
$\underline{n_\gamma \sim 1.45}$ (does not depend on E)

$\Delta y \sim \theta L \frac{\Delta E}{E}$

$E_{crit} \propto \gamma^2 B_\perp$, $\theta \propto \frac{L B_\perp}{E}$

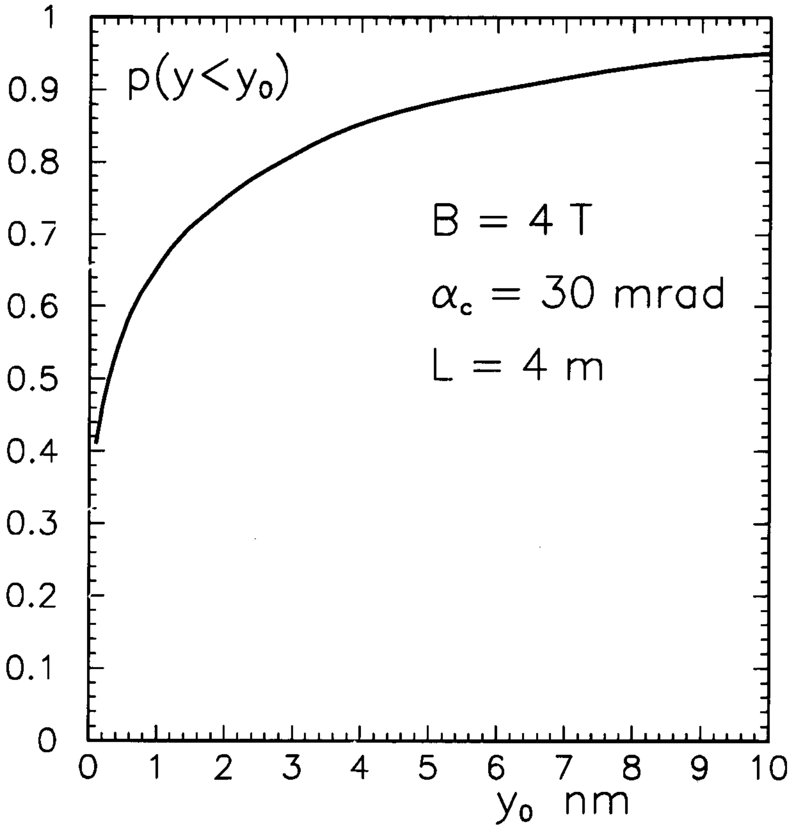
$\Rightarrow$ $\underline{\Delta y \sim L^2 B_s^2 \alpha_c^2}$ for emission of one photon (does not depend on E)

The dependence of Δy on $L B_s \alpha_c$ is even stronger due to emission of several photons

So, $\underline{\text{minimum } \sigma_y \propto L^2 B_s^2 \alpha_c^2}$ (approximatelly)

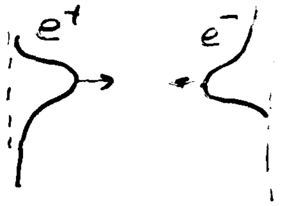
Below are results of numerical calculations.

The probability that an electron has the vertical displacement due to crab crossing $y < y_0$



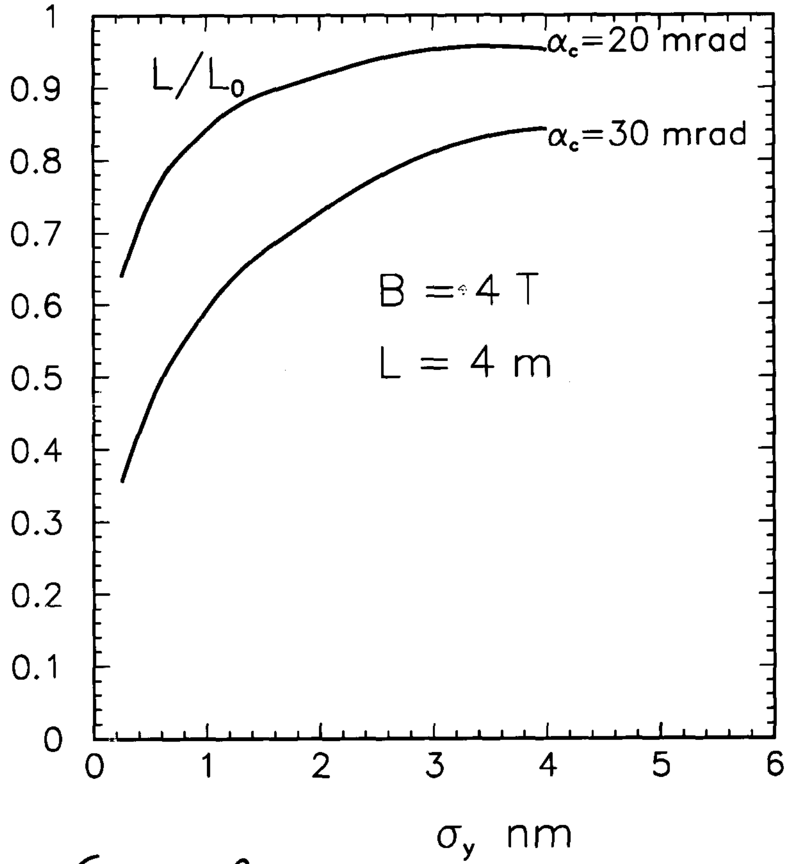
$\Rightarrow$ for given B_s, α_c, L ~35% of electrons have $\Delta y > 1$ nm

In e^+e^- collisions e^+ and e^- have opposite deflections



The luminosity will be reduced

L/L$_o$ due to crab crossing vs σ_y



For CLIC(3000) and $B_S = 4T$, $L = 4M$, $\alpha_c = 0.03$
$\sigma_y = 1nm$ and $L/L_o = 0.6$

MC simulation with account of pinch effect give $L/L_o \sim 0.7$ (for focusing to $X = 0$)

Conclusion

1) Radiation of electrons in solenoid field ($B_\perp = B_s \frac{\alpha_c}{2}$) leads to increase of the vertical beam size. The effect is a function of $B_s \alpha_c L$.

2) For realistic $B_s \alpha_c L$ the number of emitted photons $n_\gamma \sim 1$ and
$$\Delta y \propto B_s^2 \alpha_c^2 L^2$$
This lead to restriction on minimum σ_y on the level ~ 0.5 nm

3) This effect is more important for large collider energies where $\sigma_y < 1$ nm were planned, but it leads to some decrease of luminosity already at $2E_0 = 500$ GeV collider. + photon colliders

4) Radiation in quads with subsequent deflection in solenoidal field gives additional contribution

(and beam disruption angles are large)